# Quiz 4 Solutions:

1. Use cartesian coordinates to integrate the function $f(x,y) = x\sin(y)$ over the rectangle $0 \le x \le 3$, $0 \le y \le \pi$.

$$\iint x\sin(y)\,dx\,dy$$

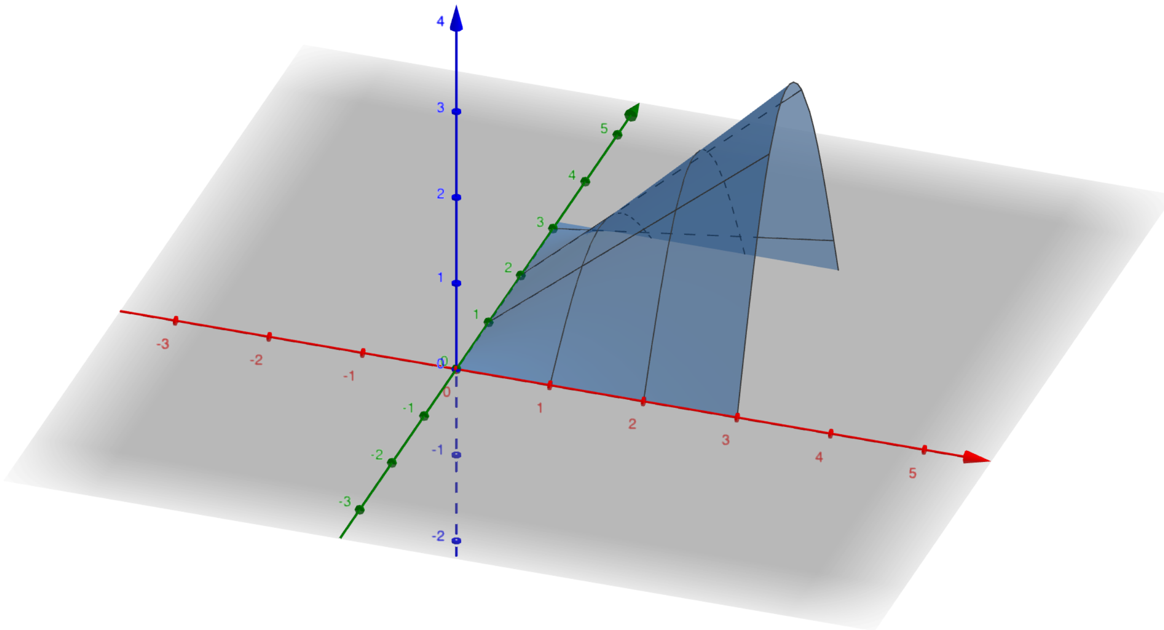
$$= \int_0^3 x\,dx \int_0^{\pi} \sin(y)\,dy$$

$$= \left(\tfrac{1}{2}x^2\right)_0^3 \cdot \left(-\cos(y)\right)_0^{\pi}$$

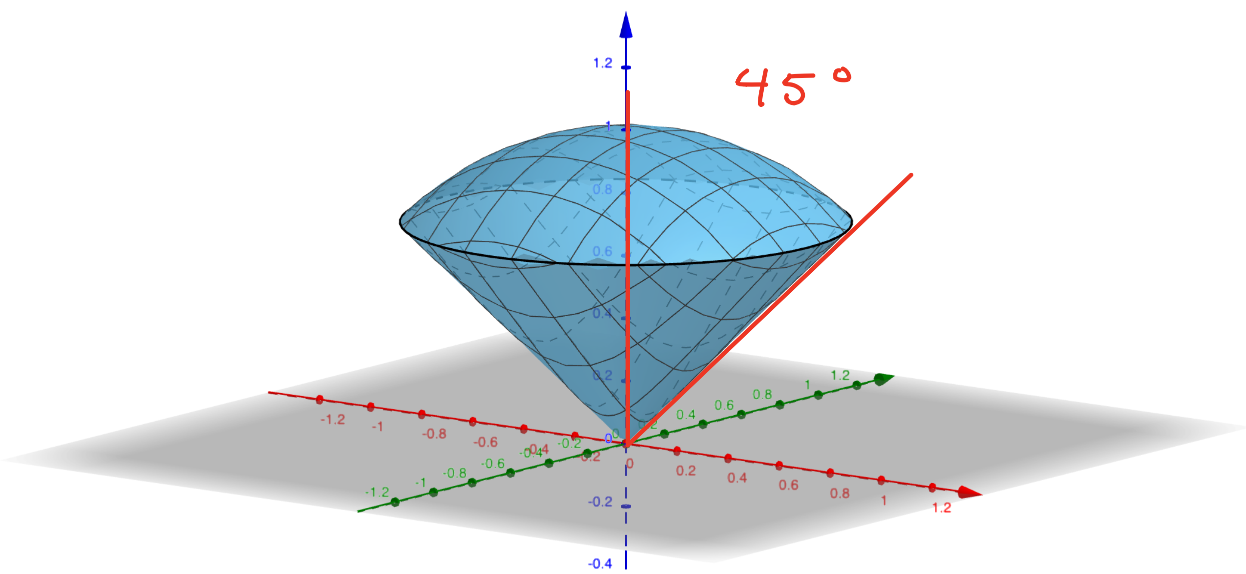
$$= \frac{1}{2}\cdot 9\left(-\underbrace{\cancel{\cos(\pi)}}_{-1} + \underbrace{\cancel{\cos(0)}}_{+1}\right)$$

$$= 9$$

If $f$ = "height" then we can think of this integral as the volume of the region above the rectangle and below the surface $z = x\sin(y)$:




2. Use spherical coordinates to find the volume of the region above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $z = \sqrt{1-x^2-y^2}$ :



we have $0 \le \theta \le 2\pi$ & $0 \le \varphi \le \pi/4$.

The sphere has radius 1 and is centered at the origin, so $0 \le \rho \le 1$.

Therefore the volume is

$$\text{vol} = \iiint dV$$

$$= \iiint \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$$

$$= \int_0^{2\pi} d\theta \int_0^1 \rho^2 d\rho \int_0^{\pi/4} \sin\varphi \, d\varphi$$

$$= 2\pi \cdot \frac{1}{3} \cdot \left(-\cos\left(\frac{\pi}{4}\right) + \cos(0)\right)$$

$$= \frac{2\pi}{3}\left(1 - \sqrt{2}/2\right)$$

[About 30% of the volume of the full hemisphere, which is $\frac{2}{3}\pi$.]