1. Consider the parametrized path:

$$\vec{r}(t) = (x(t), y(t)) = (t^2-1, t^3-t)$$

(a) Compute the velocity and speed:

$$\vec{r}\,'(t) = (x'(t), y'(t))$$
$$= (2t, 3t^2-1)$$

$$\|\vec{r}\,'(t)\| = \sqrt{(2t)^2 + (3t^2-1)^2}$$

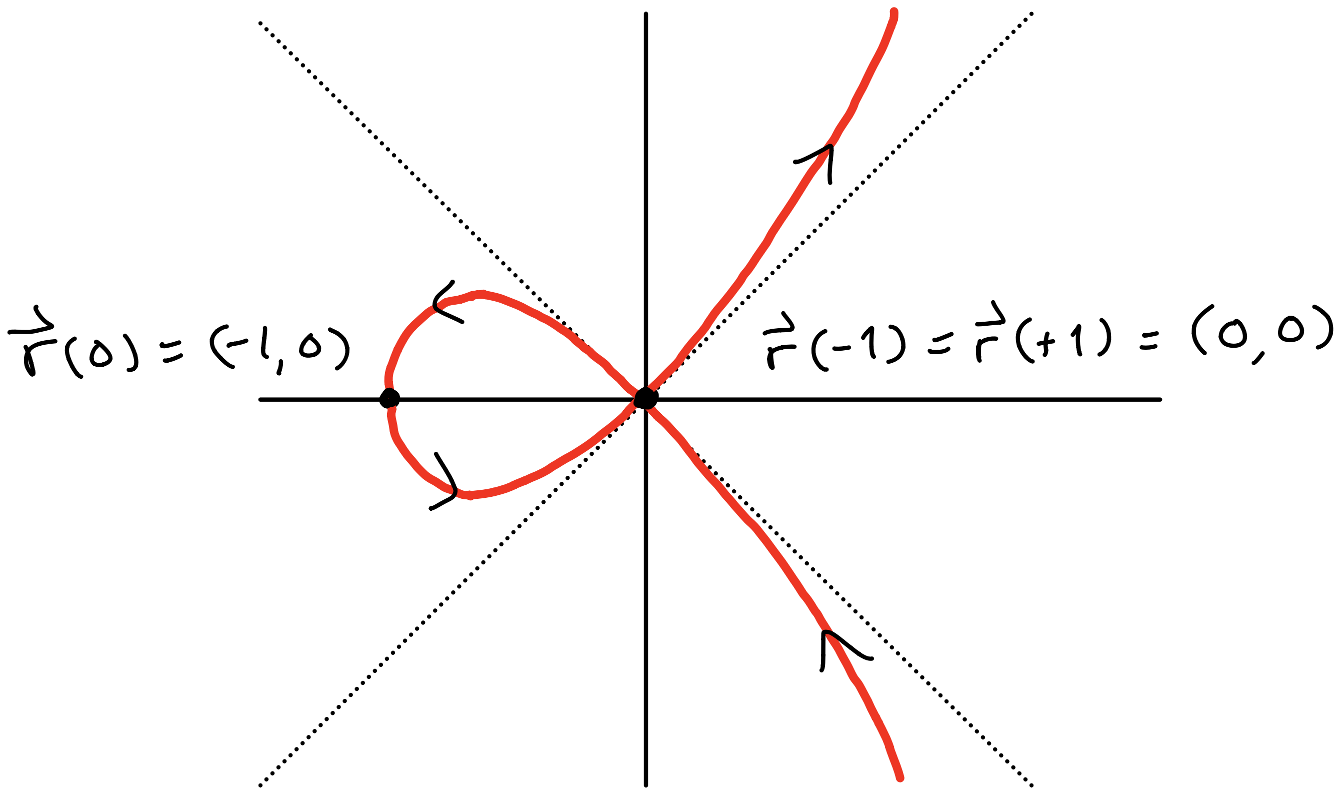
(b) Arc length between $t=-1,+1$:

$$\text{Arc length} = \int \text{speed}\, dt$$

$$= \int_{-1}^{1} \sqrt{(2t)^2 + (3t^2-1)^2}\, dt$$

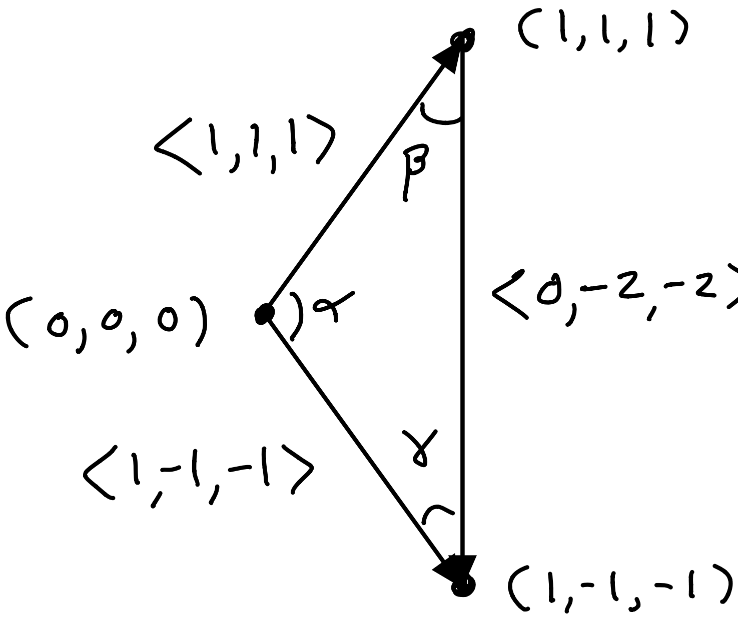
$$\approx 2.72 \quad [\text{via computer}]$$

Bonus Content: The picture of this cure is interesting



2. Consider the triangle in $\mathbb{R}^3$ with vertices

$$P = (0,0,0), \; Q = (1,1,1), \; R = (1,-1,-1)$$



(a) Compute the side lengths :

$$\|\vec{PQ}\| = \sqrt{\langle 1,1,1\rangle \cdot \langle 1,1,1\rangle}$$

$$= \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$\|\vec{PR}\| = \sqrt{\langle 1,-1,-1\rangle \cdot \langle 1,-1,-1\rangle}$$

$$= \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$\|\vec{QR}\| = \sqrt{\langle 0,-2,-2\rangle \cdot \langle 0,-2,-2\rangle}$$

$$= \sqrt{0^2+(-2)^2+(-2)^2} = \sqrt{8}$$

(b) Compute the cosines of the angles :

$$\vec{PQ} \cdot \vec{PR} = \langle 1,1,1\rangle \cdot \langle 1,-1,-1\rangle$$

$$= 1\cdot 1 + 1(-1) + 1(-1) = -1$$

$$\cos\alpha = \frac{\vec{PQ}\cdot\vec{PR}}{\|\vec{PQ}\|\,\|\vec{PR}\|} = \frac{-1}{\sqrt{3}\sqrt{3}} = -\frac{1}{3}$$

[Remark : $\alpha \approx 109.47^\circ$]

$$\vec{QP} \circ \vec{QR} = \langle -1,-1,-1\rangle \circ \langle 0,-2,-2\rangle$$

$$= (-1)\cdot 0 + (-1)(-2) + (-1)(-2) = 4$$

$$\cos\beta = \frac{\vec{QP} \circ \vec{QR}}{\|\vec{QP}\| \, \|\vec{QR}\|} = \frac{4}{\sqrt{3}\sqrt{8}} = \sqrt{2/3}$$

$$\vec{RP} \circ \vec{RQ} = \langle -1,1,1\rangle \circ \langle 0,2,2\rangle$$

$$= (-1)\cdot 0 + 1\cdot 2 + 1\cdot 2 = 4$$

$$\cos\gamma = \frac{\vec{RP} \circ \vec{RQ}}{\|\vec{RP}\| \, \|\vec{RQ}\|} = \frac{4}{\sqrt{3}\sqrt{8}} = \sqrt{2/3}$$

[Remark: $\beta = \gamma \approx 35.26^\circ$]

(c) Find the equation of the plane that contains P, Q, R. There are many possible ways to do this.

Let $(x_0, y_0, z_0) = P = (0,0,0)$.

Let $\vec{n} = \langle a,b,c\rangle = \vec{PQ} \times \vec{PR}$

$$= \langle 1,1,1\rangle \times \langle 1,-1,-1\rangle$$

$$= \langle 1(-1)-1(-1),\ 1\cdot 1-1(-1),\ 1(-1)-1\cdot 1\rangle$$

$$= \langle 0,2,-2\rangle .$$

The equation of the plane is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$0(x-0)+2(y-0)-2(z-0)=0$$

$$2y-2z=0$$

$$y-z=0$$

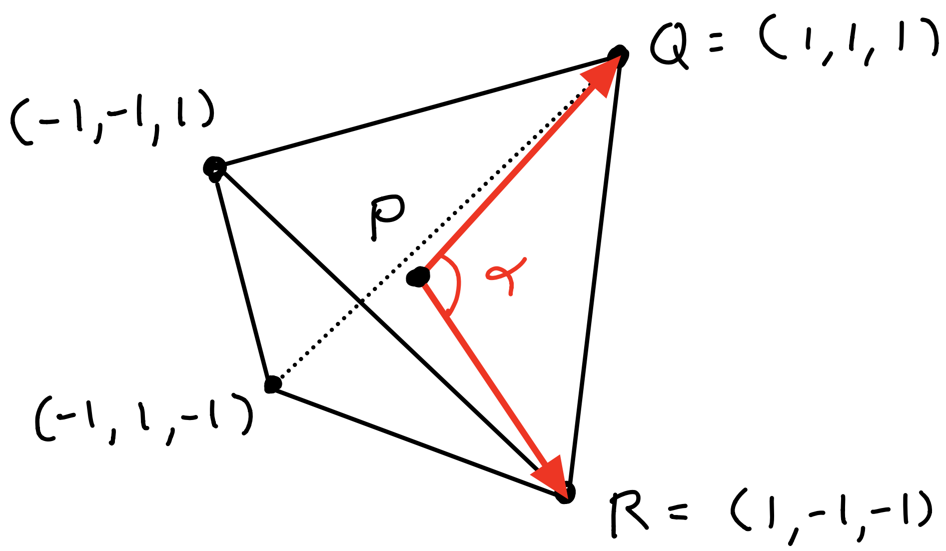
or $$y=z.$$

[Indeed, all three points $(0,0,0)$, $(1,1,1)$, $(1,-1,-1)$ satisfy $y=z$. ✓]

Bonus Content: This triangle fits inside a very interesting picture.

The 4 points $(1,1,1)$, $(-1,-1,1)$, $(1,-1,-1)$, $(-1,1,-1)$

are the vertices of a "regular tetrahedron" living in $\mathbb{R}^3$, centered at the origin $(0,0,0)$:



The angle $\alpha \approx 109.47°$ with

$$\cos \alpha = -\frac{1}{3}$$

is called the "tetrahedral angle".

Example: The angle between two Hydrogen atoms in a molecule of methane.