Math 211 **Summer 2021**
Homework 3 **Drew Armstrong**

Problem 1. Tangent Planes. For each function $f(x,y,z)$ find the equation of the tangent plane to the "level surface" $f(x,y,z)=c$ at the point (x_0,y_0,z_0), where $c=f(x_0,y_0,z_0)$.

(a) $f(x,y,z)=x^2+y^2-z/5$ and $(1,2,1)$
(b) $f(x,y,z)=x^2+y^2+z^2$ and $(x_0,y_0,z_0)=(1,-3,2)$
(c) $f(x,y,z)=(x/2)^2+(y/2)^2+(z/3)^2$ and $(2,2,3)$

Problem 2. Multivariable Chain Rule. Consider a rectangular cardboard box with length ℓ, width w and height h. If the box has no lid then the surface area is given by

$$A=\ell w+2\ell h+2wh.$$

Suppose that you measure the dimensions (in inches) to be

$$\begin{aligned}\ell&=10\pm 0.03,\\ w&=6\pm 0.02,\\ h&=3\pm 0.01.\end{aligned}$$

Use the multivariable chain rule to estimate the uncertainty in the area A.

Problem 3. Optimization. Consider the scalar field

$$f(x,y)=1+4xy-x^4-y^4.$$

Find the *critical points* (a,b) where the gradient is zero: $\nabla f(a,b)=\langle 0,0\rangle$. For each critical point, use the *second derivative test* to determine whether this point is a local maximum, local minimum, or a saddle point.

Problem 4. Directional Derivatives. Let $f(x,y)=e^{-x^2-y^2}$ represent the temperature at a point (x,y) in the plane. Suppose that you follow the path $\mathbf{r}(t)=\langle 1+t\cos\theta, 1+t\sin\theta\rangle$ with initial position $\mathbf{r}(0)=\langle 1,1\rangle$ and initial velocity $\mathbf{r}'(0)=\langle\cos\theta,\sin\theta\rangle$.

(a) Compute your rate of change of temperature when $t=0$:

$$(f\circ\mathbf{r})'(0)=\frac{d}{dt}f(\mathbf{r}(t))|_{t=0}.$$

(b) For which value of θ does the temperature increase fastest?

Problem 5. Gradient Flow. Let $f(x,y)$ be the height of a hill above the point (x,y):

$$f(x,y)=100-2x^2-y^2.$$

Suppose that you start at $(x,y)=(9,3)$ and you want to follow a path $\mathbf{r}(t)=\langle x(t),y(t)\rangle$ with $\mathbf{r}(0)=\langle 9,3\rangle$ that "always travels directly uphill," i.e., a path where the velocity $\mathbf{r}'(t)$ is always parallel to the gradient vector $\nabla f(\mathbf{r}(t))$.

(a) Show that the path $\mathbf{r}(t)=\langle 9e^{-4t},3e^{-2t}\rangle$ has this property.
(b) Show that the path $\mathbf{r}(t)=\langle (3-t)^2, 3-t\rangle$ also has this property.