Math 211 **Summer 2021**
Homework 1 **Drew Armstrong**

Problem 1. Parametrized Curves. Consider the following parametrized curves in $\mathbb{R}^2$. Eliminate t to find an equation for each curve in terms of x and y. Draw the curves.

(a) $(x,y) = (6-4t, -1+3t)$
(b) $(x,y) = (e^t, e^{2t}+1)$
(c) $(x,y) = (1+\cos t, 2+\sin t)$

[Hint: The equations for (b) and (c) might involve x^2 and y^2.]

Problem 2. A Parametrized Ellipse. The ellipse $(x/2)^2 + y^2 = 1$ can be parametrized:

$$(x,y) = (2\cos t, \sin t).$$

(a) Draw the ellipse.
(b) Compute the velocity and the speed of the parametrization.
(c) Find the equation of the tangent line to the ellipse at the point $(\sqrt{2}, \sqrt{2}/2)$. [Hint: This is the point corresponding to $t = \pi/4$. Use the velocity vector from (b) to find the slope of the tangent line. Then use the point-slope equation.]
(d) Write down an integral for the perimeter of the ellipse, and solve it using a computer. [Warning: This integral **cannot** be solved exactly using any of the standard functions.]

Problem 3. A Triangle in Space. Consider the following points in $\mathbb{R}^3$:

$$P = (1,1,0), \quad Q = (1,0,2), \quad R = (1,2,3).$$

(a) Find the coordinates of the three side vectors $\mathbf{u} = \vec{PQ}, \mathbf{v} = \vec{QR}, \mathbf{w} = \vec{PR}$.
(b) Use the length formula to compute the three side lengths $\|\mathbf{u}\|, \|\mathbf{v}\|, \|\mathbf{w}\|$.
(c) Use the dot product to compute the three angles of the triangle.

Problem 4. Some Vector Arithmetic. Let $\mathbf{u}$ and $\mathbf{v}$ be any two vectors, living in 527-dimensional space. Use the rules of vector arithmetic (pages 112 and 147) to show that

$$\|\mathbf{u}-\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(\mathbf{u} \bullet \mathbf{v}).$$

[Hint: Start with $\|\mathbf{u}-\mathbf{v}\|^2 = (\mathbf{u}-\mathbf{v}) \bullet (\mathbf{u}-\mathbf{v})$. Now use FOIL and simplify the result.]

Problem 5. Equations of Lines and Planes. The equation of the line in $\mathbb{R}^2$ that contains the point (x_0, y_0) and is perpendicular to the vector $\mathbf{n} = \langle a, b\rangle$ is

$$a(x-x_0) + b(y-y_0) = 0.$$

The equation of the plane in $\mathbb{R}^3$ that contains the point (x_0, y_0, z_0) and is perpendicular to the vector $\mathbf{n} = \langle a, b, c\rangle$ is

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0.$$

(a) Find the equation of the line containing $(-1, 2)$ and perpendicular to $\langle 3, 1\rangle$.
(b) Find the equation of the plane containing $(1,0,0)$ and perpendicular to $\langle 1, 3, -1\rangle$.
(c) Find the equation of the plane containing the points P, Q, R from Problem 3. [Hint: You can let (x_0, y_0, z_0) be any of the three points, say $(x_0, y_0, z_0) = P = (1,1,0)$. To get a vector $\mathbf{n} = \langle a, b, c\rangle$ perpendicular to the plane, let $\mathbf{n}$ be the cross product of any two vectors in the plane, say $\mathbf{n} = \mathbf{u} \times \mathbf{v}$.]