Welcome to MTH 210, Summer A, 2020.

Introduction to Linear Algebra.

Weekly HW (due Fri)	40%
Weekly Quiz (on Mon)	40%
Final Project	20%

Linear Algebra is extremely useful & getting more important every year.

Sadly, our curriculum has not kept up with this reality.

So we have one course, which is not enough time, but I will do my best to prepare you.

BALANCE

Intuition	Algebra
Geometry	Arithmetic
Pictures	Algorithms

The difficulty of the subject is to maintain connection between these two points of view.

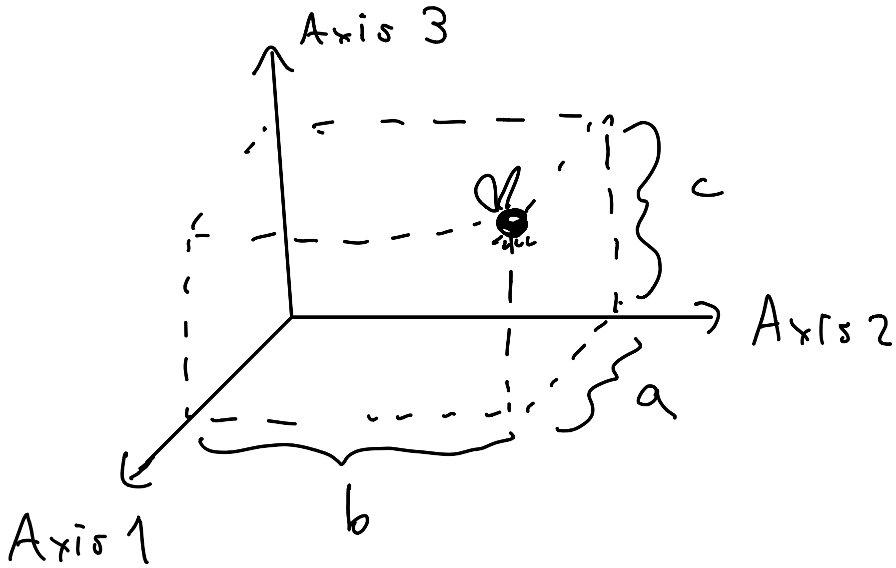
This week:

The Arithmetic of Vectors.

"Cartesian"

Idea of Coordinate Geometry:

point $\equiv$ an ordered list of numbers.



The position of the fly

$= (a, b, c)$.

Important features:

- Axes are mutually perpendicular
- dotted lines form a <u>rectangular</u> box
- The coordinates are allowed to be negative numbers.

Notation: In this course we will prefer to express points as <u>column vectors</u>

$$\vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

There is a special point called the "origin" of the coordinate system:

$$\vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

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So what?

There is an arithmetic of numbers.
Can we use coordinates to create an "arithmetic of points"?

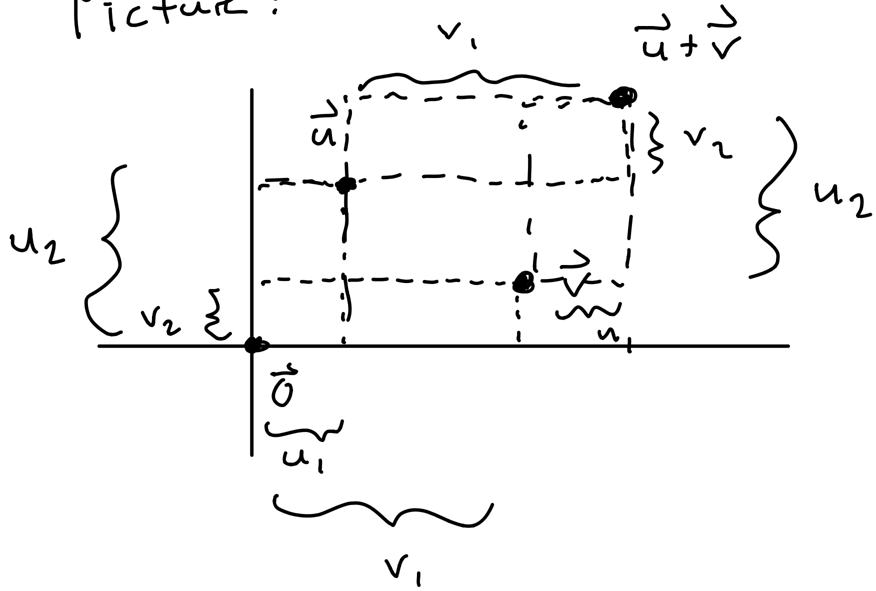
Definition: Addition of Points.

Given $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ we define

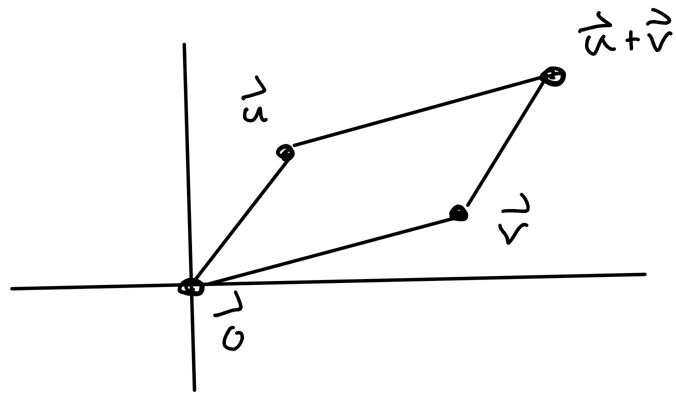
$$\vec{u} \text{ "}+\text{" } \vec{v} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix}$$

we are inventing a new operation.
But what does it mean?

Picture:



To simplify the picture we invent the concept of a <u>vector</u>.



Idea: Instead of drawing rectangular boxes, we draw the diagonals.

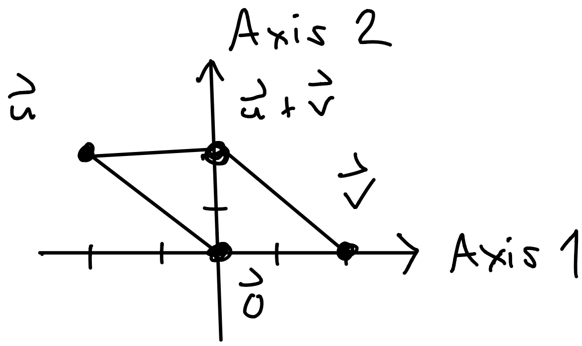
Geometric Picture of Addition:
The points $\vec{0}$, $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$ are the four vertices of a <u>parallelogram</u>.

"Parallelogram Law"

Examples:

2D. $\vec{u} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Draw points $\vec{0}$, $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$.

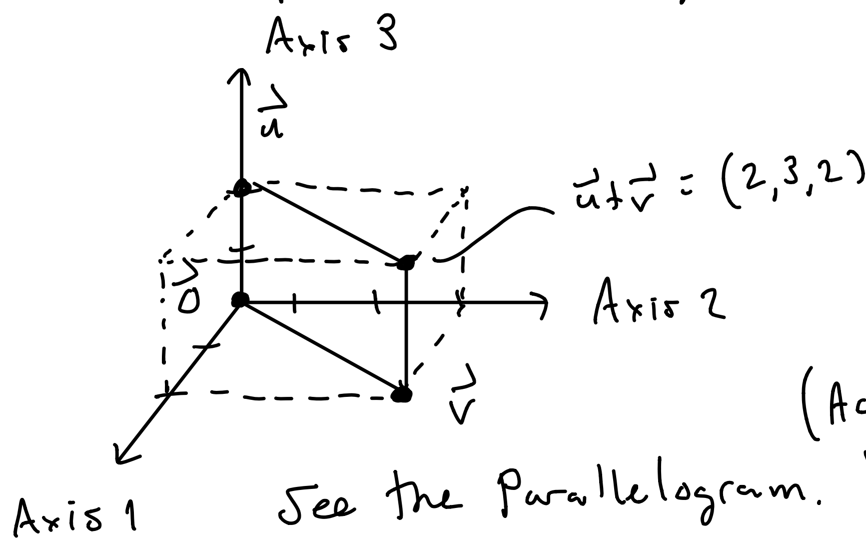


$$\vec{u}+\vec{v} = \begin{pmatrix} -2+2 \\ 2+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

See the Parallelogram?

3D: $\vec{u} = (0,0,2)$ & $\vec{v} = (2,3,0)$.

Draw the parallelogram $\vec{0}, \vec{u}, \vec{v}, \vec{u}+\vec{v}$.



See the Parallelogram. (Actually a rectangle)

Amazingly, the parallelogram rule holds in <u>any number of dimensions</u>.

However, our pictures in $n \geq 4$ dimensions will only be "impressions".

We've defined points.

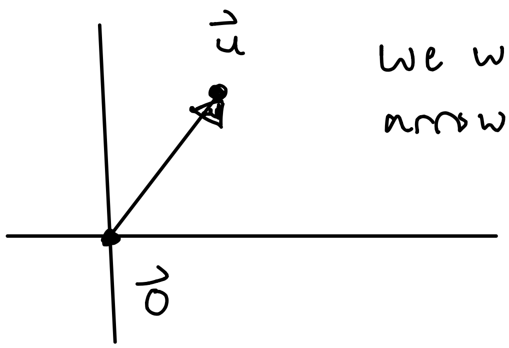
Next: what is a "vector"?

Many acceptable answers to this question:

- an ordered pair of points. (head, tail)
- a quantity with magnitude & direction.
- an element of a "vector space"

Sadly, there is no best answer.

For us, right now, each point $\vec{u}$ determines a directed line segment ("arrow") with head a point $\vec{u}$ & tail at $\vec{0}$:

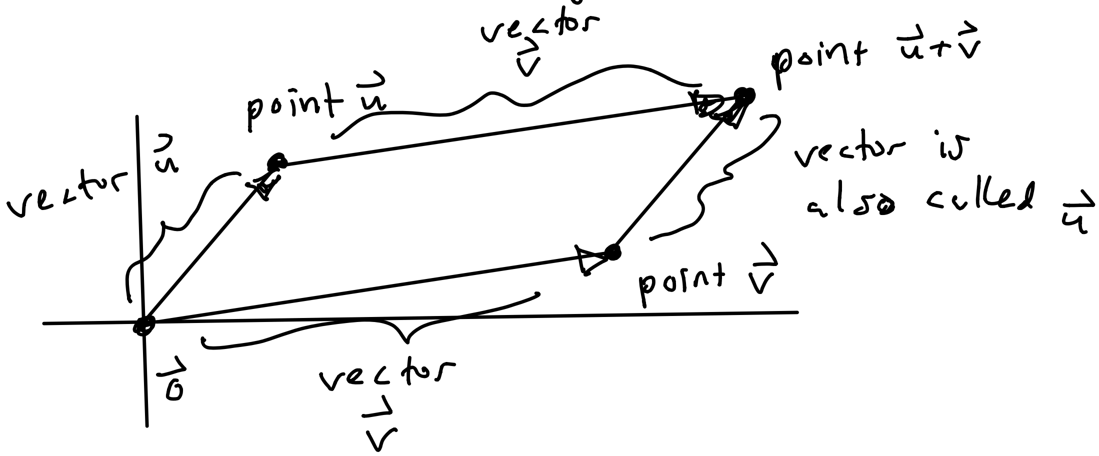


we will call this arrow "$\vec{u}$"

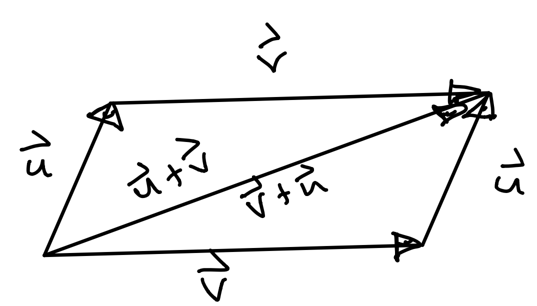
Hopefully we won't get confused by using the same symbol $\vec{u}$ for both the point and the arrow.

But this is not yet the full definition.

I say we are allowed to <u>move</u> a vector without changing its name:

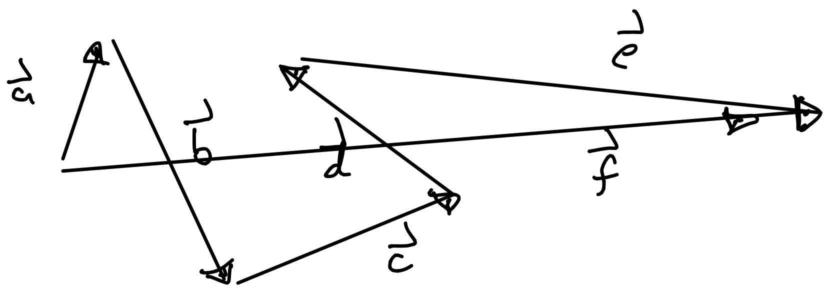


Why is this good? Because it allows us to say that "vectors add head-to-tail"



Order doesn't matter: $\vec{u}+\vec{v}=\vec{v}+\vec{u}$.

we can iterate this procedure:

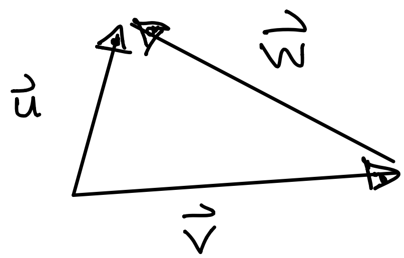


$$\vec{f} = \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e}.$$

Isaac Newton invented the mathematical abstraction of "vectors" to describe the mathematical abstraction of "forces."

From our point of view, a vector is a "movable arrow". The name of the vector = the head point of the arrow when tail is at $\vec{0}$.

Note that this idea immediately tells us how to <u>subtract</u> vectors:



Head-to-Tail Addition says

$$\vec{v} + \vec{w} = \vec{u}$$

So we might as well say

$$\vec{w} = \vec{u} \text{ "} - \text{" } \vec{v}.$$

And, in fact, this is correct if we define subtraction "componentwise"

$$\vec{u} - \vec{v} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1 - v_1 \\ u_2 - v_2 \end{pmatrix}.$$

Everything fits together nicely :)