Quiz 5 Solutions

Problem 1: a) Find the OLS best fit line $y = a + bx$ for the three data points

$$(x, y) = (-1, 0), (0, 2), (1, 1).$$

$$\begin{cases} a - b = 0 \\ a + 0b = 2 \\ a + b = 1 \end{cases}$$

no solution!

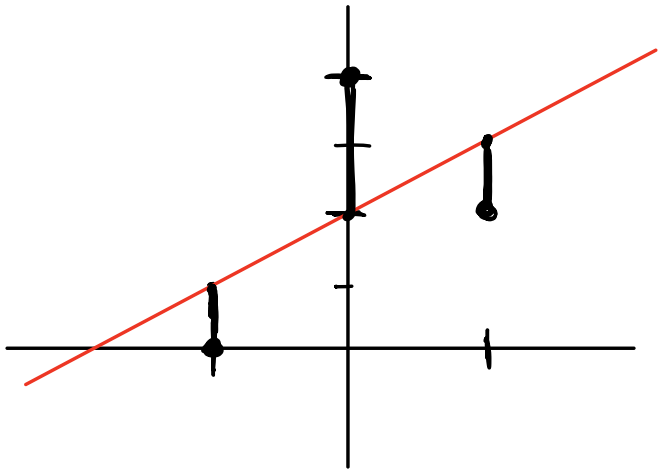
$$\begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$$

b) Draw a picture :



Sum of squares of vertical errors

$$\left\| \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right\|^2$$

is MINIMIZED.

Problem 2 : We are given

$$\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix},$$

$$\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0.5 \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Solve the recurrence

$$\vec{x}_0 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad \& \quad \vec{x}_{n+1} = \begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix} \vec{x}_n .$$

Solution: First express $\vec{x}_0$ in terms of eigenvectors:

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Then solve for $\vec{x}_n$:

$$\vec{x}_n = \begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}^n \vec{x}_0$$

$$= \begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}^n \left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}^n \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}^n \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 2 \end{pmatrix} + (0.5)^n \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Done.