

## Quiz 4 Solutions

Problem 1:

$$a) f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x \end{pmatrix}, \quad g\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x+y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x+y \end{pmatrix}$$

$$[f] = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad [g] = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

$$b) [f \circ g] = [f][g]$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Problem 2:

$$a) \left( \begin{array}{ccc|ccc} \textcircled{1} & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$\left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \textcircled{1} & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array} \right)$$

$$\left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right) \text{ Done.}$$

$$\text{So } \left( \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{array} \right)^{-1} = \left( \begin{array}{ccc} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{array} \right).$$

$$\text{b) } \left( \begin{array}{ccc} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right)^{-1} = \left( \begin{array}{ccc} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{array} \right)$$

$$\text{because } (A^T)^{-1} = (A^{-1})^T.$$