Quiz 3 Solutions

Problem 1:

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 4 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Problem 2:

(a) $$\begin{cases} x + 2y + z = 0, \\ x + 2y + 2z = 0, \\ 2x + 4y + z = 0. \end{cases}$$

From Problem 1 we have

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 2 & 2 & 0 \\ 2 & 4 & 1 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

So the system is equivalent to

$$\begin{cases} x + 2y = 0, \\ z = 0. \end{cases}$$

Solution :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2y \\ y \\ 0 \end{pmatrix} = y \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}.$$

(b) The nullspace $N(A)$ is just the solution to part (a).

So $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ is a basis.

(c) A basis for the column space is given by the "pivot columns" 1 & 3. (The "nonpivot column 2 is redundant.) So a basis is

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \ \& \ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

The column space is a plane

$$C(A) = \left\{ s \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right\}$$