Quiz 1 Solutions

Problem 1:

(a): $\vec{u} \cdot \vec{u} = 3^2 + 1^2 = 10$

$\vec{v} \cdot \vec{v} = 1^2 + 2^2 = 5$

$\vec{u} \cdot \vec{v} = 3 + 2 = 5$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\sqrt{\vec{u} \cdot \vec{u}}\,\sqrt{\vec{v} \cdot \vec{v}}} = \frac{5}{\sqrt{10}\,\sqrt{5}} \quad \left(= \frac{1}{\sqrt{2}} \right)$$

[Hence $\theta = 45°$]

(b): $(\vec{x} + \vec{y}) \cdot (\vec{x} + \vec{y})$

$= \vec{x} \cdot \vec{x} + 2\vec{x} \cdot \vec{y} + \vec{y} \cdot \vec{y}$

$= 1 + 2(0) + 1 = 2.$

$(\vec{x} - 2\vec{y}) \cdot (\vec{x} - 2\vec{y})$

$= \vec{x} \cdot \vec{x} - 4\vec{x} \cdot \vec{y} + 4\vec{y} \cdot \vec{y}$

$= 1 - 4(0) + 4 = 5$

$(\vec{x} + \vec{y}) \cdot (\vec{x} - 2\vec{y})$

$= \vec{x} \cdot \vec{x} - 1\,\vec{x} \cdot \vec{y} - 2\vec{y} \cdot \vec{y}$

$$= 1 - 3(0) - 2 = -1$$

Hence

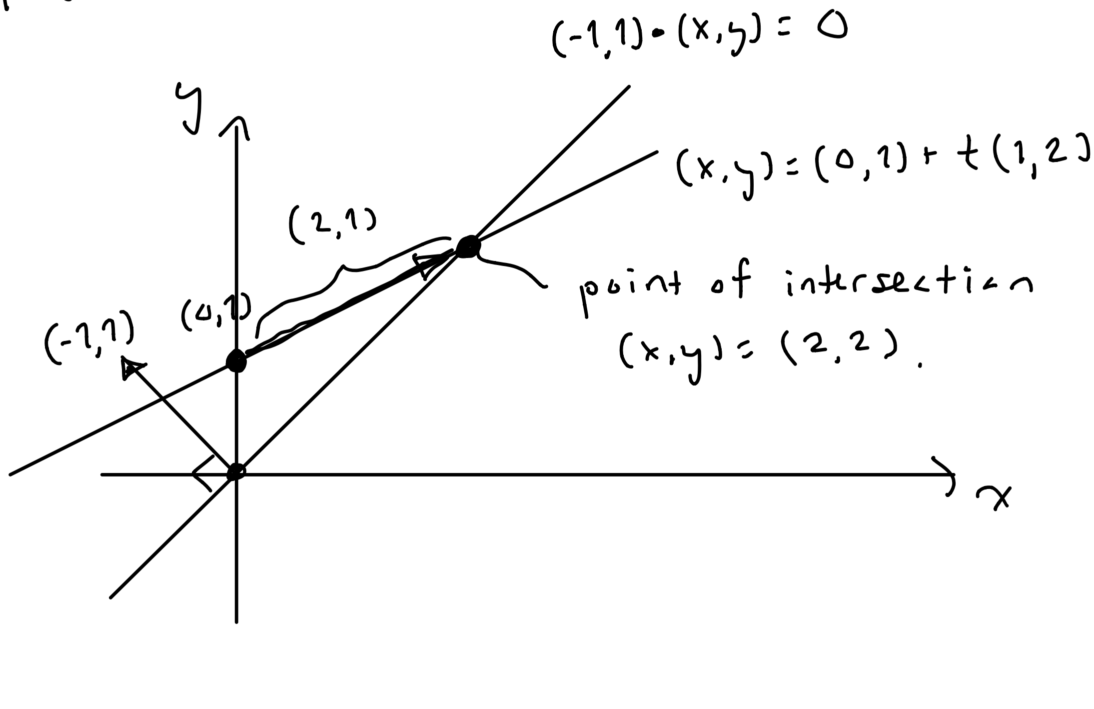
$$\cos\theta = \frac{-1}{\sqrt{2}\sqrt{5}}$$

$$[\theta \approx 108.43^\circ]$$

Remark: Since $\vec{x}$ & $\vec{y}$ are "orthonormal" you get the same result by thinking

$$\vec{x} + \vec{y} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \& \quad \vec{x} - 2\vec{y} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Problem 2:



Computation:

Substitute

$$(x, y) = (0,1) + t(2,1)$$
$$= (2t, 1+t)$$

into the equation $-x + y = 0$ to get

$$-(2t) + (1+t) = 0$$
$$1 - t = 0$$
$$t = 1.$$

Hence $(x, y) = (2t, 1+t)$

$$= (2(1), 1+(1)) = (2,2).$$