

## Quiz 1 Solutions

Problem 1:

$$(a) : \vec{u} \cdot \vec{u} = 3^2 + 1^2 = 10$$

$$\vec{v} \cdot \vec{v} = 1^2 + 2^2 = 5$$

$$\vec{u} \cdot \vec{v} = 3 + 2 = 5$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\sqrt{\vec{u} \cdot \vec{u}} \sqrt{\vec{v} \cdot \vec{v}}} = \frac{5}{\sqrt{10} \sqrt{5}} \left( = \frac{1}{\sqrt{2}} \right)$$

$$[\text{Hence } \theta = 45^\circ]$$

$$\begin{aligned} (b) : & (\vec{x} + \vec{y}) \cdot (\vec{x} + \vec{y}) \\ &= \vec{x} \cdot \vec{x} + 2\vec{x} \cdot \vec{y} + \vec{y} \cdot \vec{y} \\ &= 1 + 2(a) + 1 = 2. \end{aligned}$$

$$\begin{aligned} (\vec{x} - 2\vec{y}) \cdot (\vec{x} - 2\vec{y}) \\ &= \vec{x} \cdot \vec{x} - 4\vec{x} \cdot \vec{y} + 4\vec{y} \cdot \vec{y} \\ &= 1 - 4(a) + 4 = 5 \end{aligned}$$

$$\begin{aligned} (\vec{x} + \vec{y}) \cdot (\vec{x} - 2\vec{y}) \\ &= \vec{x} \cdot \vec{x} - 1\vec{x} \cdot \vec{y} - 2\vec{y} \cdot \vec{y} \end{aligned}$$

$$= 1 - 3(0) - 2 = -1$$

Hence

$$\cos \theta = \frac{-1}{\sqrt{2} \sqrt{5}}$$

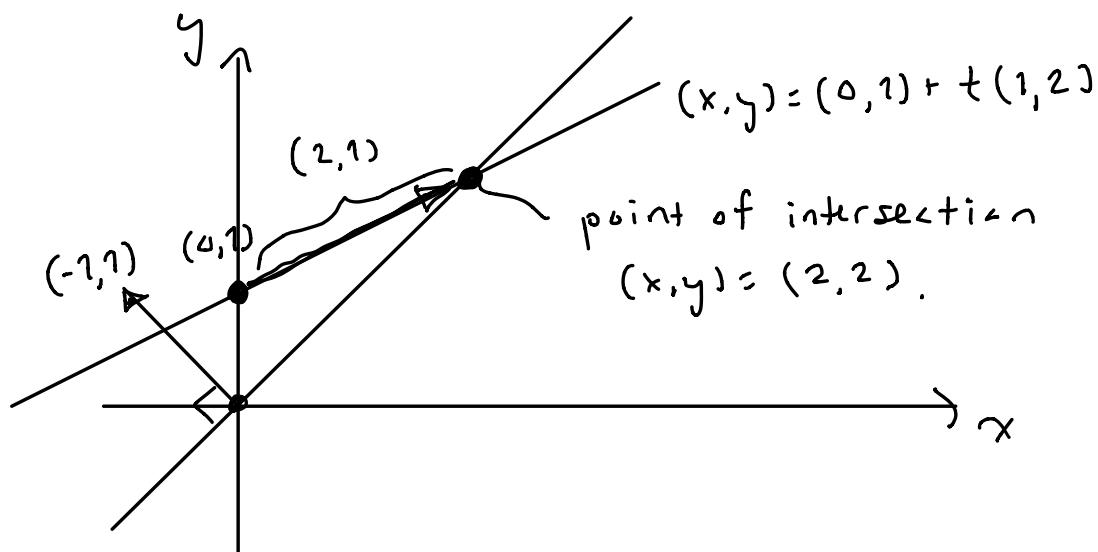
$$[\theta \approx 108.43^\circ]$$

Remark: Since  $\vec{x}$  &  $\vec{y}$  are "orthonormal"  
you get the same result by thinking

$$\vec{x} + \vec{y} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ & } \vec{x} - 2\vec{y} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Problem 2 :

$$(-1,1) \cdot (x,y) = 0$$



Computation:

Substitute

$$\begin{aligned}(x, y) &= (0, 1) + t(2, 1) \\ &= (2t, 1+t)\end{aligned}$$

into the equation  $-x + y = 0$  to get

$$\begin{aligned}-2t + (1+t) &= 0 \\ 1 - t &= 0 \\ t &= 1.\end{aligned}$$

Hence  $(x, y) \in (2t, 1+t)$

$$\begin{aligned}&= (2(1), 1+(1)) = (2, 2).\end{aligned}$$