Intro to Linear Algebra
Quiz 1

Summer 2017
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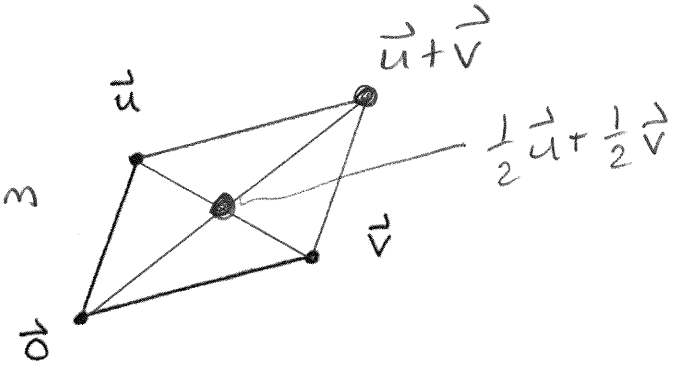
Problem 1. Consider two points $\vec{u}$ and $\vec{v}$ in the Cartesian plane.

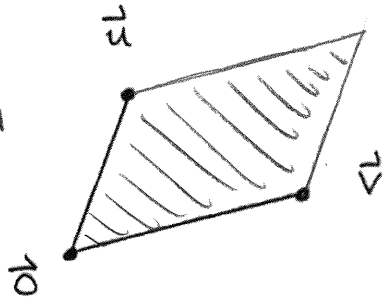
(a) Draw the points $\vec{u}+\vec{v}$ and $\frac{1}{2}\vec{u}+\frac{1}{2}\vec{v}$:

vertex and midpoint of the parallelogram



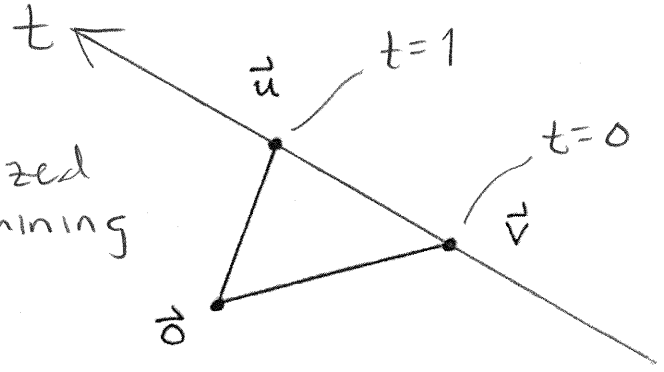
(b) Draw the set of all points $a\vec{u}+b\vec{v}$ where $0 \leqslant a \leqslant 1$ and $0 \leqslant b \leqslant 1$:

filled parallelogram



(c) Draw the set of all points $t\vec{u}+(1-t)\vec{v}$ where t ranges over all numbers:

parametrized line containing $\vec{u}$ & $\vec{v}$



Problem 2. Consider two vectors $\vec{v}$ and $\vec{w}$ with the following properties:

$$\vec{v} \bullet \vec{v} = \|v\|^2 = 1, \qquad \vec{w} \bullet \vec{w} = \|w\|^2 = 2 \qquad \text{and} \qquad \vec{v} \bullet \vec{w} = 0.$$

(a) Compute the **length** of the vector $\vec{v} + \vec{w}$.

$$\|\vec{v}+\vec{w}\|^2 = (\vec{v}+\vec{w})\circ(\vec{v}+\vec{w})$$
$$= \vec{v}\circ\vec{v} + 2\vec{v}\circ\vec{w} + \vec{w}\circ\vec{w}$$
$$= 1 + 2(0) + 2 = 3$$

$$\Longrightarrow \quad \|\vec{v}+\vec{w}\| = \sqrt{3}$$

(b) Compute the **length** of the vector $\vec{v} - 2\vec{w}$.

$$\|\vec{v}-2\vec{w}\|^2 = (\vec{v}-2\vec{w})\circ(\vec{v}-2\vec{w})$$
$$= \vec{v}\circ\vec{v} - 4\vec{v}\circ\vec{w} + 4\vec{w}\circ\vec{w}$$
$$= 1 - 4(0) + 4(2) = 9$$

$$\Longrightarrow \quad \|\vec{v}-2\vec{w}\| = \sqrt{9} = 3.$$

(c) Compute the **cosine of the angle** between the vectors $\vec{v} + \vec{w}$ and $\vec{v} - 2\vec{w}$.

First: $(\vec{v}+\vec{w})\circ(\vec{v}-2\vec{w}) = \vec{v}\circ\vec{v} - \vec{v}\circ\vec{w} - 2\vec{w}\circ\vec{w}$
$= 1 - 0 - 2(2) = -3.$

Then:

$$\cos\theta = \frac{(\vec{v}+\vec{w})\circ(\vec{v}-2\vec{w})}{\|\vec{v}+\vec{w}\|\cdot\|\vec{v}-2\vec{w}\|} = \frac{-3}{\sqrt{3}\cdot 3} = \frac{-1}{\sqrt{3}}$$