Math 210 **Summer 2020**
Homework 2 **Drew Armstrong**

Problem 1. Parallel Lines. Consider the vector $\mathbf{a} = (1, 2)$.

(a) Draw the 5 lines $\mathbf{a} \bullet \mathbf{x} = b$ for the values $b \in \{-2, -1, 0, 1, 2\}$.
(b) Draw the 5 points $\mathbf{x} = (b/\|\mathbf{a}\|^2)\mathbf{a}$ for the same values $b \in \{-2, -1, 0, 1, 2\}$.

Problem 2. Perpendicular and Parallel Lines. Consider two lines in the x, y-plane:

$$ax + by = c \quad \text{and} \quad a'x + b'y = c'.$$

(a) Find an equation involving the constants a, b, c, a', b', c' to determine when the lines are **perpendicular**. [Hint: Recall that vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ are perpendicular when $\mathbf{u} \bullet \mathbf{v} = u_1v_1 + u_2v_2 = 0$.]
(b) Find an equation involving the constants a, b, c, a', b', c' to determine when the lines are **parallel**. [Hint: Recall that vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ are parallel when $\mathbf{u} = t\mathbf{v}$, or $(u_1, u_2) = (tv_1, tv_2)$ for some constant t.]

Problem 3. Intersection of Two Lines. Consider the following system of two linear equations in the two unknowns x and y (where c is a constant):

$$\begin{cases} x + 2y = 2, \\ x + cy = 0. \end{cases}$$

(a) Solve for x and y in the case $c = 1$. Draw a picture of your solution.
(b) For which value of c does the system have **no solution**? Draw a picture in this case.

Problem 4. Intersection of Two Planes and the Cross Product. For any two vectors $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ in $\mathbb{R}^3$ we define the *cross product* as follows:

$$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1).$$

(a) Use algebra to verify the identities $\mathbf{u} \bullet (\mathbf{u} \times \mathbf{v}) = 0$ and $\mathbf{v} \bullet (\mathbf{u} \times \mathbf{v}) = 0$. It follows that the vector $\mathbf{u} \times \mathbf{v}$ is simultaneously perpendicular to $\mathbf{u}$ and $\mathbf{v}$.
(b) Use the cross product to solve the following system of linear equations:

$$\begin{cases} x + y + 2z = 0, \\ 2x + y + 3z = 0. \end{cases}$$

[Hint: The solution is a line of the form $\mathbf{x} = t\mathbf{a}$ or $(x, y, z) = (ta, tb, tc)$, where a, b, c are some constants and t is a free parameter.]

Problem 5. Intersection of Three Planes. Consider the following system of 3 linear equations in the 3 unknowns x, y, z (where c is a constant):

$$\begin{cases} x + y + 2z = 0, \\ 2x + y + 3z = 0, \\ 2x + 3y + cz = 4. \end{cases}$$

(a) Solve for x, y, z when $c = 1$. In this case the three planes intersect at a unique point. [Hint: The intersection of the first two planes is a line $(x, y, z) = (ta, tb, tc)$ from Problem 2(b). Plug this into the third equation and solve for t.]
(b) For which value of c does the system have **no solution**? In this case the third plane is parallel to—and does not contain—the line of intersection of the first two planes. [Hint: Try to solve as in part (a). Look for a value of c that makes this impossible.]