Math 210 Homework 1 — Summer 2020 Drew Armstrong

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Drew Armstrong

Problem 1. Drawing Points. Consider the vectors $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (1, 2)$.

(a) Draw the 9 points (x, y), where $x, y \in \{0, 1, 2\}$.
(b) Add the point $(1.5, 1.75)$ to your picture from (a).
(c) Draw the 9 points $x\mathbf{u} + y\mathbf{v}$, where $x, y \in \{0, 1, 2\}$.
(d) Add the point $1.5\mathbf{u} + 1.75\mathbf{v}$ to your picture from (c).

Problem 2. Drawing Lines. Consider the same vectors $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (1, 2)$.

(a) Add the lines $\{(x, y) : x + y = 1\}$ and $\{(x, y) : x - 2y = -2\}$ to your picture from 1(a).
(b) I claim that each of the following set of points is a line: $\{x\mathbf{u} + y\mathbf{v} : x + y = 1\}$ and $\{x\mathbf{u} + y\mathbf{v} : x - 2y = -2\}$. Add these lines to your picture from 1(c).

Problem 3. Shading Regions. Keep $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (1, 2)$.

(a) Draw the following shaded regions:
$\{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$,
$\{(x, y) : x \geq 1\}$,
$\{(x, y) : x \leq 1 \text{ and } y \leq 1\}$.
(b) Draw the following shaded regions:
$\{x\mathbf{u} + y\mathbf{v} : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$,
$\{x\mathbf{u} + y\mathbf{v} : x \geq 1\}$,
$\{x\mathbf{u} + y\mathbf{v} : x \leq 1 \text{ and } y \leq 1\}$.

Problem 4. The Angle Between Vectors. Let $\mathbf{x}, \mathbf{y}$ be two vectors with the same number of components and let θ be the angle between them. The generalized Pythagorean theorem tells us that $\mathbf{x} \bullet \mathbf{y} = \|\mathbf{x}\|\|\mathbf{y}\| \cos\theta$.

(a) First let $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (1, 2)$. Use the Pythagorean theorem to compute the angle between $\mathbf{x} = 2\mathbf{u} + \mathbf{v} = (7, 4)$ and $\mathbf{y} = \mathbf{u} + \mathbf{v} = (4, 3)$.
(b) Now let $\mathbf{u}$ and $\mathbf{v}$ be any vectors in 100-dimensional space satisfying $\mathbf{u}\bullet\mathbf{v} = 5$, $\mathbf{u}\bullet\mathbf{u} = 10$ and $\mathbf{v} \bullet \mathbf{v} = 5$. Use the Pythagorean theorem and the rules of vector arithmetic to compute the angle between $\mathbf{x} = 2\mathbf{u} + \mathbf{v}$ and $\mathbf{y} = \mathbf{u} + \mathbf{v}$.

Problem 5. The General Equation of a Line. If a, b, c are constant then the equation $ax + by = c$ represents a line in the x, y-plane. This equation can also be expressed as

$$\mathbf{a} \bullet \mathbf{x} = c,$$

where $\mathbf{a} = (a, b)$ and $\mathbf{x} = (x, y)$.

(a) Draw the line $\mathbf{a} \bullet \mathbf{x} = -2$ when $\mathbf{a} = (1, -2)$.
(b) Show that the line $\mathbf{a} \bullet \mathbf{x} = c$ is perpendicular to the vector $\mathbf{a}$. [Hint: If $\mathbf{x}_1 = (x_1, y_1)$ and $\mathbf{x}_2 = (x_2, y_2)$ are any two points on the line, show that $\mathbf{a} \bullet (\mathbf{x}_1 - \mathbf{x}_2) = 0$.]
(c) Show that the line $\mathbf{a} \bullet \mathbf{x} = c$ contains the point $\mathbf{x} = (c/\|\mathbf{a}\|^2)\mathbf{a}$.