Math 210 Homework 2

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Problem 1.

(a) Draw the following three parallel lines in the Cartesian plane:

$$x + 2y = -5, \quad x + 2y = 0, \quad x + 2y = 5.$$

(b) Fill in the blanks: The equation $ax + by = c$, or in other words

$$\begin{pmatrix} a \\ b \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix} = c,$$

represents a line in the Cartesian plane that is perpendicular to the vector _______ and contains the point _______ . [Hint: There are infinitely many correct answers. For the second blank, try to find the point of the form $(x, y) = t(a, b)$ that is on this line.]

(c) Fill in the blank: The lines $ax + by = c$ and $a'x + b'y = c'$ are perpendicular to each other if and only if _______ .

Problem 2.

(a) Draw the circle $x^2 + y^2 = 25$ in the Cartesian plane.

(b) Compute the intersection of this circle with the line $4x + 3y = 0$.

(c) Draw the two lines that are tangent to the circle at the points of intersection found in part (b). Find the equations of these two lines. [Hint: The tangent lines are both perpendicular to the line $4x + 3y = 0$.]

Problem 3.

(a) Solve for x and y in the following vector equation:

$$x \begin{pmatrix} 1 \\ -3 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

(b) Draw a picture of your solution using head-to-toe vector addition.

(c) Draw a picture of your solution as an intersection of two lines, one perpendicular to the vector $(1, 1)$ and one perpendicular to the vector $(-3, 1)$. Find the equations of these two lines.

Problem 4.

(a) Compute the intersection of the planes

$$x + y + z = 0 \quad \text{and} \quad x + 2y + 3z = 0$$

as a "parametrized line" in Cartesian space. [Hint: Let $z = t$ be a "free parameter".]

(b) Use your answer from part (a) to find some vector (x, y, z) that is simultaneously perpendicular to both $(1, 1, 1)$ and $(1, 2, 3)$. [Hint: The answer is not unique.]

(c) Now compute the intersection of the line from part (a) with the third plane

$$-x + 2y + 4z = 2.$$

(d) Finally, compute the solution of the following vector equation:

$$x \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + y \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + z \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}.$$