Math 210 F **Spring 2016**
Homework 4 **Drew Armstrong**

Problem 1. Two Pictures of the Matrix Notation. Consider the following:

$$A = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & -2 & 0 \end{pmatrix} \qquad \text{and} \qquad \vec{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$$

(a) Compute $A\vec{x}$ as a **linear combination** of the columns of A.
(b) Compute $A\vec{x}$ by taking the **dot product** of $\vec{x}$ with the rows of A.

Problem 2. Finding Implicitly Defined Matrices.

(a) Find the matrices I, P, and R such that for all x and y we have

$$I\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad P\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}, \qquad \text{and} \qquad R\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}.$$

(b) Find the matrix A such that for all x, y, and z we have

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+y \\ x+2z \end{pmatrix}$$

Problem 3. Discovering the Matrix Product. Consider the following:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & 1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \text{and} \qquad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

(a) Compute the vector $\vec{v} = B\vec{x}$.
(b) Now compute the vector $A\vec{v} = A(B\vec{x})$.
(c) Finally, find the matrix C such that for all x and y we have $C\vec{x} = A(B\vec{x})$. Can you think of a good name for this matrix?

Problem 3. Working With Abstract Matrices. Recall that a "matrix of shape $m \times n$" has m rows and n columns. If A is $m \times n$ and B is $n \times r$ then the matrix product AB is defined and has shape $m \times r$. If the number of columns of A does **not** equal the number of rows of B then the product AB is **not defined**.

(a) Suppose A has shape 3×5, B has shape 5×3, and C has shape 3×2. Which of the following matrices are defined, and what are their shapes?

$$AB, \quad BA, \quad ABC, \quad CBA, \quad C^TBA.$$

(b) Now let A and B be have arbitrary shape. Answer the following as true or false:
- If $A^2(= AA)$ is defined then A is square.
- If A^TA is defined then A is square.
- If $AB = B$ then A is square.
- If $AB = B$ then B is square.
- If AB and BA are both defined then A and B are both square.
- If AB and BA are both defined then AB and BA are both square.

Problem 5. Matrix Multiplication is Not (Generally) Commutative.

(a) Compute the matrix product on both sides to show that

$$\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} \neq \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}.$$

(b) Find all values of a, b, c, d such that

$$\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}a & b\\c & d\end{pmatrix}.$$