Math 210 F **Spring 2016**
Homework 1 **Drew Armstrong**

Problem 1.

(a) Draw the cube with corners at

$$\begin{pmatrix}0\\0\\0\end{pmatrix},\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix},\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix},\begin{pmatrix}1\\1\\1\end{pmatrix}.$$

(b) Draw the triangle in 3D with corners

$$\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}$$

and compute the values of its angles using the dot product.

Problem 2. Let $\vec{u}=\begin{pmatrix}1\\2\end{pmatrix}$ and $\vec{v}=\begin{pmatrix}3\\1\end{pmatrix}$.

(a) Draw the points $\vec{u}$ and $\vec{v}$ together with $\frac{1}{2}\vec{u}+\frac{1}{2}\vec{v}$, $\frac{3}{4}\vec{u}+\frac{1}{4}\vec{v}$, $\frac{1}{4}\vec{u}+\frac{1}{4}\vec{v}$ and $\vec{u}+\vec{v}$.

(b) Draw the infinite line of points $a\vec{u}+b\vec{v}$ where a and b are any numbers (positive or negative) satisfying $a+b=1$. [Hint: Draw two points from the line and connect them.]

(c) Draw the infinite line of points $a\vec{u}+a\vec{v}$ where a is any number (positive or negative). [Same hint as part (b).]

(d) Shade the finite region of the plane containing the points $a\vec{u}+b\vec{v}$ where $0\leq a\leq 1$ and $0\leq b\leq 1$. What is this shape?

(e) Shade the infinite region of the plane containing the points $a\vec{u}+b\vec{v}$ where $0\leq a$ and $0\leq b$.

Problem 3. Recall that the dot product of two n-dimensional vectors $\vec{u}=(u_1,u_2,\ldots,u_n)$ and $\vec{v}=(v_1,v_2,\ldots,v_n)$ is defined by

$$\vec{u}\bullet\vec{v}:=u_1v_1+u_2v_2+\cdots+u_nv_n.$$

For any three n-dimensional vectors $\vec{u},\vec{v},\vec{w}$ and any number a, show that

$$\vec{u}\bullet(\vec{v}+a\vec{w})=(\vec{u}\bullet\vec{v})+a(\vec{u}\bullet\vec{w}).$$

This tells us that the dot product has properties similar to multiplication of numbers.

Problem 4. Let $\vec{u},\vec{v}$ be two n-dimensional vectors (I won't tell you what n is) and assume that they both have unit length: $\|\vec{u}\|=\|\vec{v}\|=1$.

(a) Compute the dot products $\vec{u}\bullet(-\vec{u})$, $(\vec{u}+\vec{v})\bullet(\vec{u}-\vec{v})$, and $(\vec{u}-2\vec{v})\bullet(\vec{u}+2\vec{v})$.

(b) Now assume that we also have $\vec{u}\bullet\vec{v}=0$ (i.e., the vectors $\vec{u}$ and $\vec{v}$ are perpendicular). In this case, compute the angle between the vectors $\vec{u}+2\vec{v}$ and $3\vec{u}+\vec{v}$.