Hello!

Welcome to MTH 210:
Intro to Linear Algebra.

Linear algebra is important!
Getting more important every year.
Unfortunately, math curriculum is kind of stale (~1960s Sputnik era).

Today: Data & Algorithms require linear algebra.
We need 2 semesters of linear algebra but right now we only have 1.
Oh well...

BALANCE

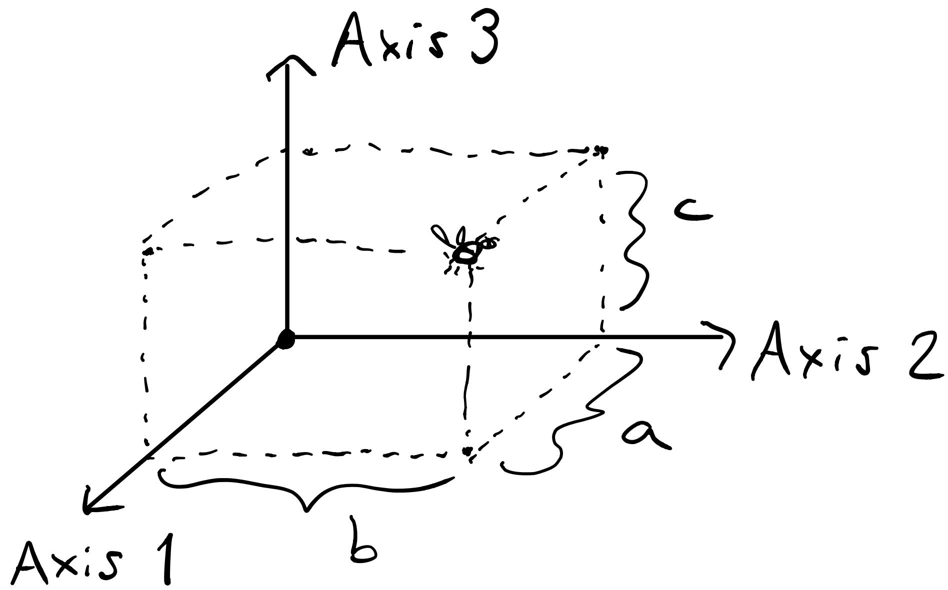
Intuition	Algebra
Geometry	Arithmetic
Pictures	Algorithms.

The difficulty is to maintain connection between these points of view.

First Topic:
The arithmetic of vectors.

What is a "vector"?

Descartes, 1637.



Position of the fly = (a, b, c)

"Cartesian coordinates"

This leads to an idea:

a point in space = an ordered triple of real numbers.

Geometry $\longleftrightarrow$ Arithmetic

Notation: In this course we will express cartesian coordinates as "column vectors"

$$\vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

point = ordered triple of numbers

There is a special point called the "origin" of the coordinate system:

$$\vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

[A bit strange because the real world does not have a special point.

Einstein: There is no **best** coordinate system. So, later (soon) we will have to discuss "change of coordinates."]

Numbers can be added.

Can we use this to "add points"?

Given two points $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ we define their "sum" as follows:

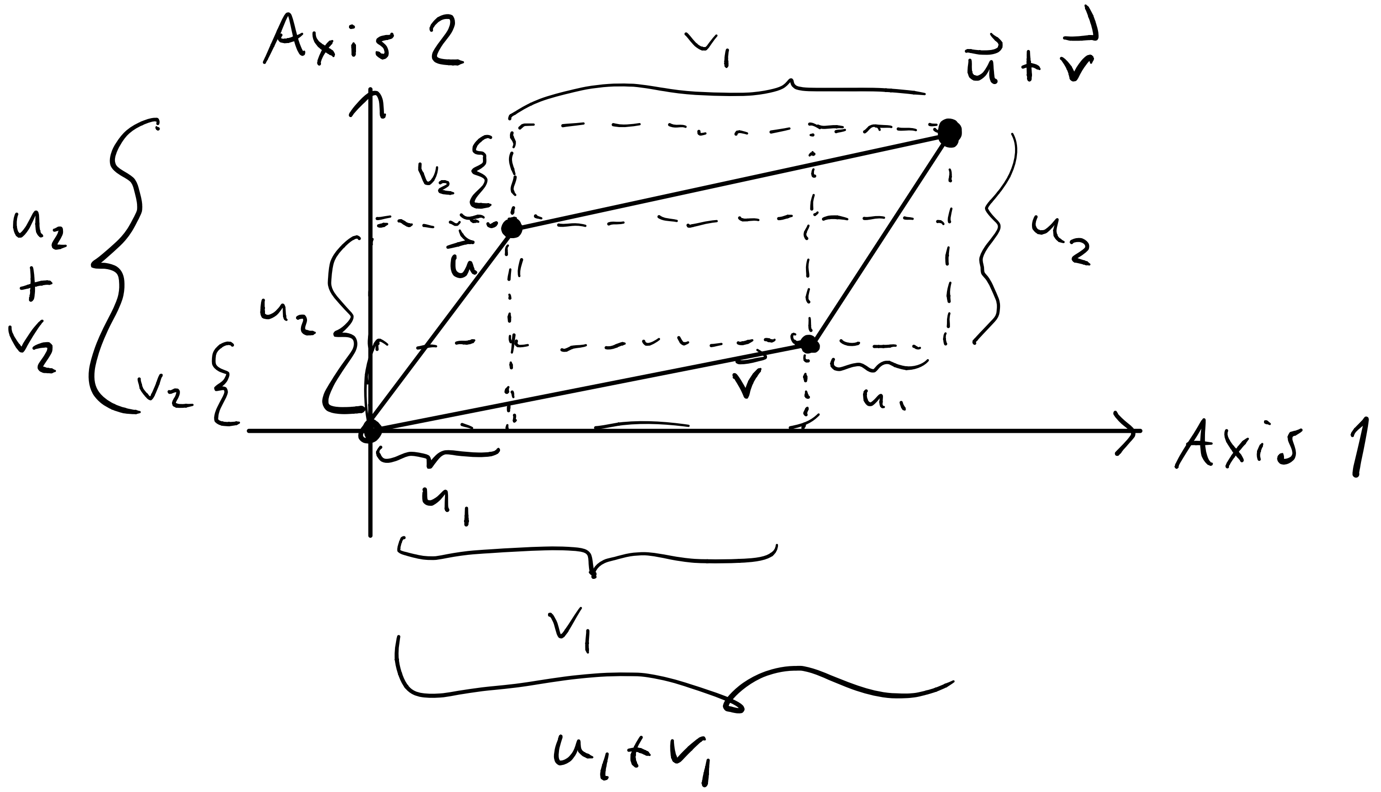
$$\text{``}\vec{u} + \vec{v}\text{''} := \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix}.$$

Symbol borrowed from Pascal programming language, "is defined to be"

This definition is obvious, but what does it **mean**?

$$\text{point} + \text{point} = ?$$

Picture:

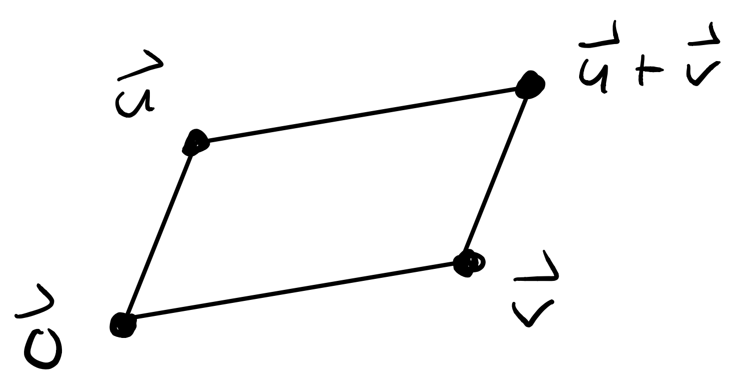


Key Observation: The 4 points

$\vec{0}, \vec{u}, \vec{v}, \vec{u}+\vec{v}$

are the vertices of a <u>parallelogram</u>.

"point + point" $\rightsquigarrow$ parallelogram.



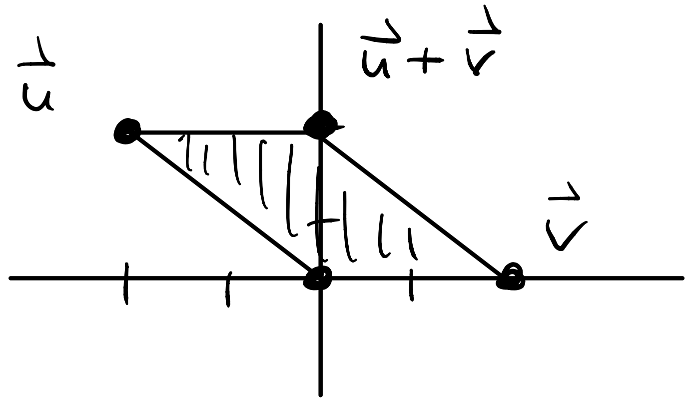
The same idea works in 3D but it is harder to draw.

Examples:

2D: $\vec{u} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Algebra: $\vec{u} + \vec{v} = \begin{pmatrix} -2+2 \\ 2+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

Geometry:

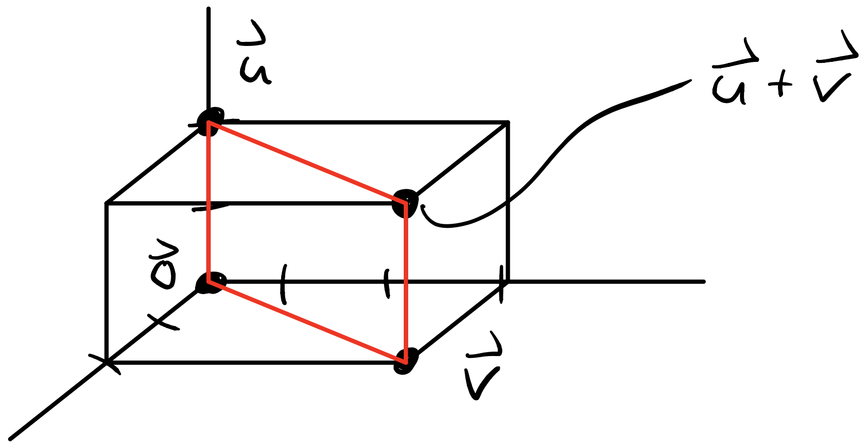


parallelogram!

3D: $\vec{u} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$.

Algebra: $\vec{u} + \vec{v} = \begin{pmatrix} 0+2 \\ 0+3 \\ 2+0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$

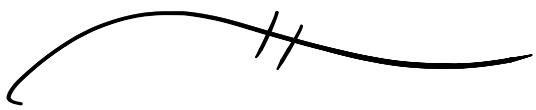
Geometry:



Parallelogram! (Actually rectangle)

Amazingly, the "parallelogram law" holds in any number of dimensions.

Even in 4D? Yes.

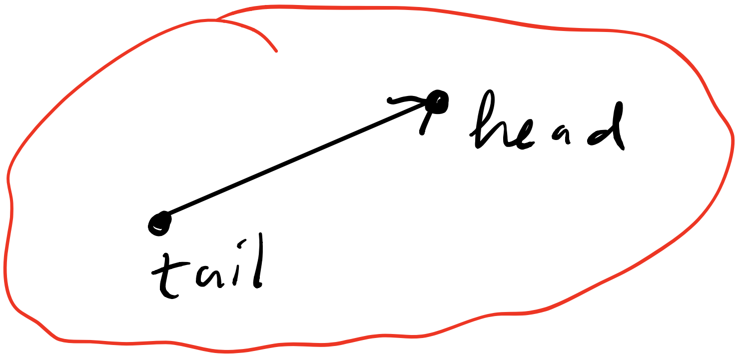


We've defined <u>points</u>.

Next: What is a <u>vector</u>?

- something with magnitude & direction
- an ordered pair of points (head, tail),

which can be thought of as an arrow:



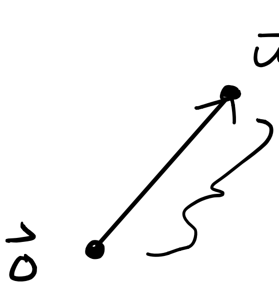
Intuition

- Fancy: A vector is "an element of a vector space."

These are all correct, there is no <u>best</u> answer.

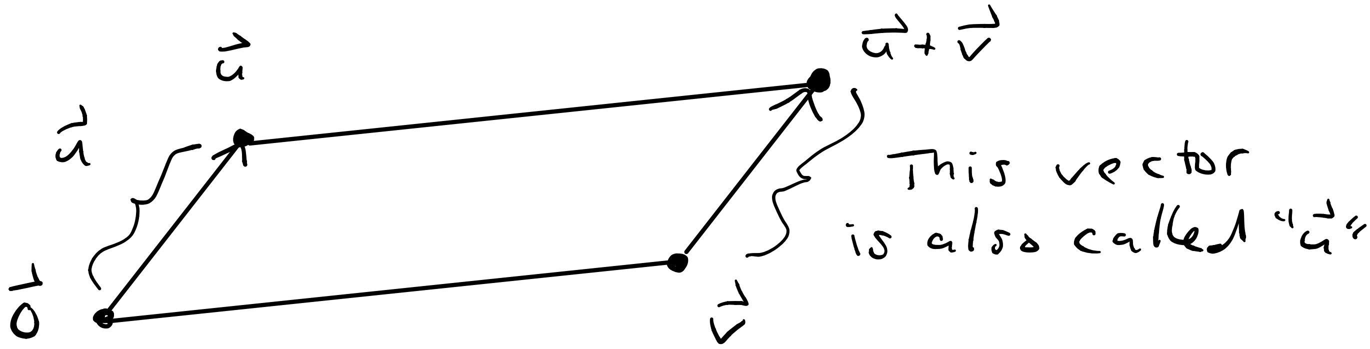
Points vs. Vectors ?

Notation:



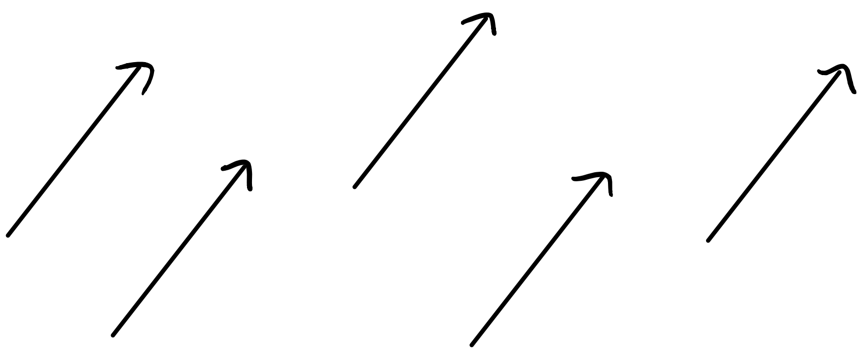
This vector is called "$\vec{u}$"

We are also allowed to move the arrow:



A vector can be moved around, as long as we don't change its <u>length</u> or <u>direction</u>.

<u>Subtle</u>: Two arrows can look different but have the same name "$\vec{u}$." There are ∞ many arrows with this name!



all called "$\vec{u}$"

That's it for today.