Math 210 Quiz 2

Fall 2020
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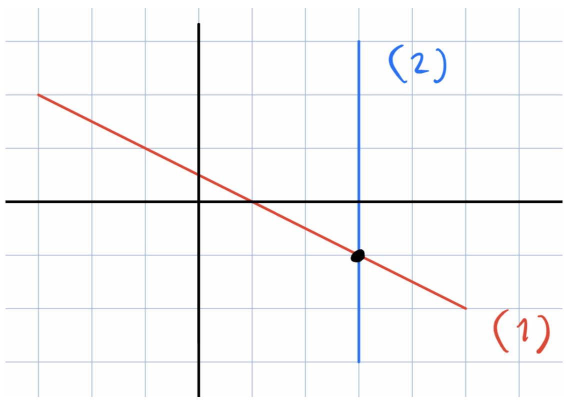
You have 20 minutes to write the quiz. No collaboration is allowed. When finished, you have 5 minutes to upload a pdf scan of your work to the google classroom.

Problem 1. [4 points] Consider the following system of two lines in $\mathbb{R}^2$:

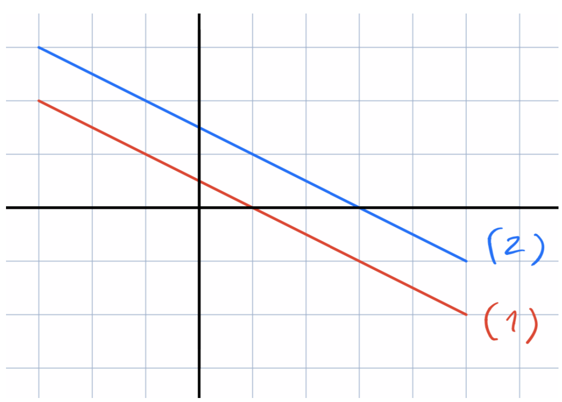
$$\begin{array}{c} (1) \\ (2) \end{array} \left\{ \begin{array}{rcrcl} x & + & 2y & = & 1, \\ 2x & + & cy & = & 6. \end{array} \right.$$

(a) Find the point of intersection when $c = 0$.
(b) For which value of c are the two lines **parallel** (i.e., have no point of intersection)?

(a): If $c = 0$ then equation (2) becomes $2x = 6$ and hence $x = 3$. Then substituting into (1) gives $3 + 2y = 1$ and hence $y = -1$. We conclude that the point of intersection is $(x, y) = (3, -1)$. Picture:



(b): Recall that lines $ax + by = c$ and $a'x + b'y = c'$ are parallel if and only if $a'b = ab'$. If (1) and (2) are parallel then we must have $1 \cdot c = 2 \cdot 2$, hence $c = 4$. Picture:



Alternatively, we can try to solve the system by elimination. If (1) and (2) have a common solution then the equation $(3) = (2) - 2(1)$ is also has a solution:

$$(3) : (c - 4)y = 4.$$

But this equation has no solution when $c = 4$.

Problem 2. **[6 points]** Consider the following system of three planes in $\mathbb{R}^3$:

$$\begin{array}{c} (1) \\ (2) \\ (3) \end{array} \left\{ \begin{array}{ccccccc} x & + & 0 & + & 2z & = & 1, \\ 0 & + & y & - & z & = & 2, \\ x & + & 2y & + & cz & = & 0. \end{array} \right.$$

(a) Find a parametrization for the line of intersection of (1) and (2). [Hint: Let $z = t$.]
(b) Find the point of intersection of (1),(2),(3) when $c = 5$. [Hint: Solve for t.]
(c) For which value of c does the system have **no solution**?

(a): By setting $z = t$ in (1) and (2) we obtain the line of intersection:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1-2t \\ 2+t \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}.$$

(b): To compute the point of intersection of (1),(2),(3) when $c = 5$, we substitute the parametrized line from part (a) into the plane (3) to obtain

$$\begin{aligned} x + 2y + 5z &= 0 \\ (1-2t) + 2(2+t) + 5(t) &= 0 \\ 5 + 5t &= 0 \\ t &= -1. \end{aligned}$$

Hence the point of intersection is $(x, y, z) = (1 - 2t, 2 + t, t) = (3, 1, -1)$.

(c): If we try to intersect the line from (a) with the plane (3) when c is general, then we obtain

$$\begin{aligned} x + 2y + cz &= 0 \\ (1-2t) + 2(2+t) + c(t) &= 0 \\ 5 + ct &= 0 \\ ct &= -5. \end{aligned}$$

This equation has no solution when $c = 0$.