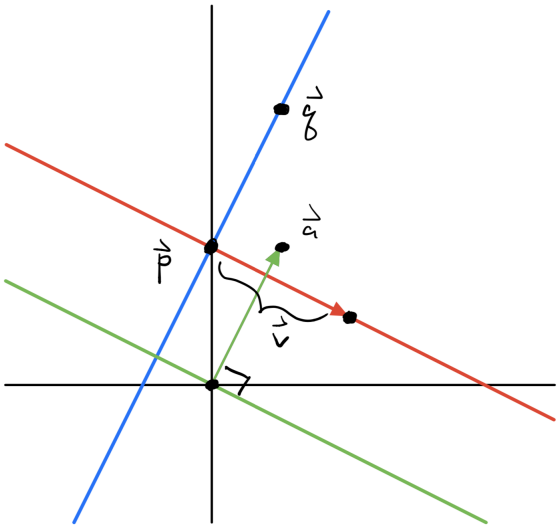
Math 210 **Fall 2020**
Quiz 1 **Drew Armstrong**

You have 20 minutes to write the quiz. No collaboration is allowed. When finished, you have 5 minutes to take a picture/scan of your quiz and upload to the google classroom.

Problem 1. [6 points]

(a) Draw the line $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ where $\mathbf{p} = (0, 2)$ and $\mathbf{v} = (2, -1)$.
(b) Draw the line $\mathbf{x} = t\mathbf{p} + (1-t)\mathbf{q}$ where $\mathbf{p} = (0, 2)$ and $\mathbf{q} = (1, 4)$.
(c) Draw the line $\mathbf{a} \bullet \mathbf{x} = 0$ where $\mathbf{a} = (1, 2)$.

Picture: (a) is red, (b) is blue, (c) is green.



Problem 2. [4 points]

(a) Compute the cosine of the angle between $\mathbf{u} = (1, 1)$ and $\mathbf{v} = (-1, 2)$.
(b) Compute the cosine of the angle between $\mathbf{x} + \mathbf{y}$ and $\mathbf{x} - \mathbf{y}$, where

$$\mathbf{x} \bullet \mathbf{x} = \mathbf{y} \bullet \mathbf{y} = 1 \quad \text{and} \quad \mathbf{x} \bullet \mathbf{y} = 0.$$

(a): We have $\mathbf{u} \bullet \mathbf{u} = 1^2 + 1^2 = 2$, $\mathbf{v} \bullet \mathbf{v} = (-1)^2 + 2^2 = 5$ and $\mathbf{u} \bullet \mathbf{v} = 1(-1) + 1(2) = 1$, hence

$$\cos\theta = \frac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\mathbf{u} \bullet \mathbf{v}}{\sqrt{\mathbf{u} \bullet \mathbf{u}}\sqrt{\mathbf{v} \bullet \mathbf{v}}} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}}.$$

(b): We have

$$(\mathbf{x} + \mathbf{y}) \bullet (\mathbf{x} + \mathbf{y}) = \mathbf{x} \bullet \mathbf{x} + 2\mathbf{x} \bullet \mathbf{y} + \mathbf{y} \bullet \mathbf{y} = 1 + 2(0) + 1 = 2,$$
$$(\mathbf{x} - \mathbf{y}) \bullet (\mathbf{x} - \mathbf{y}) = \mathbf{x} \bullet \mathbf{x} - 2\mathbf{x} \bullet \mathbf{y} + \mathbf{y} \bullet \mathbf{y} = 1 - 2(0) + 1 = 2,$$
$$(\mathbf{x} + \mathbf{y}) \bullet (\mathbf{x} - \mathbf{y}) = \mathbf{x} \bullet \mathbf{x} - \mathbf{y} \bullet \mathbf{y} = 1 - 1 = 0,$$

and hence

$$\cos\theta = \frac{(\mathbf{x} + \mathbf{y}) \bullet (\mathbf{x} - \mathbf{y})}{\|\mathbf{x} + \mathbf{y}\|\|\mathbf{x} - \mathbf{y}\|} = \frac{0}{\sqrt{2}\sqrt{2}} = 0.$$

See lecture notes for discussion.