Math 210 **Fall 2020**
Homework 2 **Drew Armstrong**

Problem 1. Planes in Space. (Answers are not unique.)

(a) Express the plane $\mathbf{x} = (0,0,1) + s(1,1,1) + t(1,2,3)$ in the form $ax + by + cz = d$.
(b) Express the plane $x + 2y + 4z = 6$ in the form $\mathbf{x} = \mathbf{p} + s\mathbf{u} + t\mathbf{v}$.

Problem 2. Perpendicular and Parallel Lines. Consider two lines in the plane:

$$ax + by = c \quad \text{and} \quad a'x + b'y = c'.$$

(a) Find an equation involving a, b, c, a', b', c' to determine when the lines are **perpendicular**. [Hint: Recall that two vectors $\mathbf{u} = (u, v)$ and $\mathbf{u}' = (u', v')$ are perpendicular if and only if $\mathbf{u} \bullet \mathbf{u} = uu' + vv' = 0$.]
(b) Find an equation involving a, b, c, a', b', c' to determine when the lines are **parallel**. [Hint: Recall that two vectors $\mathbf{u} = (u, v)$ and $\mathbf{u}' = (u', v')$ are parallel if and only if $\mathbf{u}' = t\mathbf{u}$ for some nonzero constant t.]

Problem 3. Intersection of Two Lines. Consider the following system of two linear equations in the two unknowns x and y (where c is a constant):

$$\begin{cases} x + 3y = 6, \\ 2x + cy = 0. \end{cases}$$

(a) Solve for x and y in the case $c = -3$. Draw a picture of your solution.
(b) For which value of c does the system have **no solution**? Draw a picture in this case.

Problem 4. Intersection of Two Planes (Cross Product). For any two vectors $\mathbf{u} = (u, v, w)$ and $\mathbf{u}' = (u', v', w')$ in $\mathbb{R}^3$ we define the *cross product* as follows:

$$\mathbf{u} \times \mathbf{u}' = (vw' - v'w, u'w - uw', uv' - u'v) \in \mathbb{R}^3.$$

(a) Use algebra to verify the identities $\mathbf{u} \bullet (\mathbf{u} \times \mathbf{u}') = 0$ and $\mathbf{u}' \bullet (\mathbf{u} \times \mathbf{u}') = 0$. It follows that the vector $\mathbf{u} \times \mathbf{u}'$ is simultaneously perpendicular to $\mathbf{u}$ and $\mathbf{u}'$.
(b) Use the cross product to solve the following system of linear equations:

$$\begin{cases} x + y + 2z = 0, \\ 3x + 4y + 5z = 0. \end{cases}$$

[Hint: The solution is a line $(x, y, z) = t(u, v, w)$ where the vector (u, v, w) is parallel to both planes, i.e., is simultaneously perpendicular to $(1, 1, 2)$ and $(3, 4, 5)$.]

Problem 5. Intersection of Three Planes. Consider the following system of 3 linear equations in the 3 unknowns x, y, z (where c is a constant):

$$\begin{cases} x + y + 2z = 0, \\ 3x + 4y + 5z = 0, \\ x + 2y + cz = -2. \end{cases}$$

(a) Solve for x, y, z when $c = 4$. In this case the three planes intersect at a unique point. [Hint: The intersection of the first two planes is the line $(x, y, z) = t(u, v, w)$ from 2(b). Substitute this into the third plane and solve for t.]
(b) For which value of c does the system have **no solution**? In this case the third plane is parallel to—and does not contain—the line of intersection of the first two planes. [Hint: Try to solve as in part (a). Look for a value of c that makes this impossible.]