Math 210 **Fall 2020**
Homework 1 **Drew Armstrong**

Problem 1. Coordinate Systems.

(a) Draw the 9 points (x, y) where $x, y \in \{0, 1, 2\}$. Draw the lines $y = -x+1$ and $y = 2x-2$. Shade the region where $1 \leq x \leq 2$ and $0 \leq y \leq 2$.

(b) Now let $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (-1, 2)$. Draw the 9 points $x\mathbf{u} + y\mathbf{v}$ where $x, y \in \{0, 1, 2\}$. Draw the lines $\{x\mathbf{u} + y\mathbf{v} : y = -x + 1\}$ and $\{x\mathbf{u} + y\mathbf{v} : y = 2x - 2\}$. Shade the region $\{x\mathbf{u} + y\mathbf{v} : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2\}$.

Problem 2. Midpoints. Consider the same points $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (-1, 2)$.

(a) Draw the lines $t\mathbf{u} + (1 - t)\mathbf{v}$ and $s(\mathbf{u} + \mathbf{v})$. Show that $(\mathbf{u} + \mathbf{v})/2$ is the intersection point of these lines. Explain the geometric meaning.

(b) Now consider $\mathbf{w} = (2, 4)$. Draw the points $\mathbf{u}, \mathbf{v}, \mathbf{w}$. Also draw the point $(\mathbf{u} + \mathbf{v} + \mathbf{w})/3$ and explain its geometric meaning.

Problem 3. The Angle Between Vectors. The general Pythagorean Theorem tells us that for any vectors $\mathbf{u}, \mathbf{v}$ in any number of dimensions we have $\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$ measured tail-to-tail.

(a) Compute the angle between $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (-1, 2)$.

(b) Now let $\mathbf{u}$ and $\mathbf{v}$ be any two vectors in 10 dimensional space satisfying $\mathbf{u} \bullet \mathbf{u} = 10$, $\mathbf{v} \bullet \mathbf{v} = 5$ and $\mathbf{u} \bullet \mathbf{v} = -1$. Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.

(c) Now let $\mathbf{x}$ and $\mathbf{y}$ be any two vectors in 100 dimensional space satisfying $\mathbf{x} \bullet \mathbf{x} = \mathbf{y} \bullet \mathbf{y} = 1$ and $\mathbf{x} \bullet \mathbf{y} = 0$. Compute the angle between $\mathbf{u} = 3\mathbf{x} + \mathbf{y}$ and $\mathbf{v} = -\mathbf{x} + 2\mathbf{y}$.

Problem 4. A Line in the Plane. Let $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (-1, 2)$.

(a) Find the Cartesian equation of the line $t\mathbf{u} + (1 - t)\mathbf{v}$, i.e., in terms of x and y.

(b) Express this equation in the form $\mathbf{a} \bullet (x, y) = c$ for some vector $\mathbf{a}$ and some constant c.

Problem 5. Parallel Lines. If a, b, c are constant then the equation $ax + by = c$ represents a line in the plane. This equation can also be expressed as

$$\mathbf{a} \bullet \mathbf{x} = c,$$

where $\mathbf{a} = (a, b)$ and $\mathbf{x} = (x, y)$.

(a) Draw the 5 lines $\mathbf{a} \bullet \mathbf{x} = c$ where $\mathbf{a} = (1, 2)$ and $c \in \{-2, -1, 0, 1, 2\}$.

(b) Now let $\mathbf{a} = (a, b)$ be an arbitrary nonzero vector in the plane and let c be an arbitrary constant. Prove that the line $\mathbf{a} \bullet \mathbf{x} = c$ is perpendicular to the line $t\mathbf{a}$. [Hint: For any two points $\mathbf{x}_1$ and $\mathbf{x}_2$ on the first line, show that the vector $\mathbf{x}_1 - \mathbf{x}_2$ is perpendicular to the vector $t\mathbf{a}$ for any t, i.e., that $(t\mathbf{a}) \bullet (\mathbf{x}_1 - \mathbf{x}_2) = 0$.]

It follows that any two lines of the form $\mathbf{a} \bullet \mathbf{x} = c_1$ and $\mathbf{a} \bullet \mathbf{x} = c_2$ are **parallel** (i.e., because they are both perpendicular to the line $t\mathbf{a}$).