Math 210 **Fall 2019**
Homework 3 **Drew Armstrong**

Problem 1. In class I stated that a system of linear equations has either 0, 1, or ∞ many solutions. Let's examine this claim.

(a) Suppose that $\mathbf{x}_0 = (x_0, y_0, z_0)$ and $\mathbf{x}_1 = (x_1, y_1, z_1)$ are two solutions to the linear equation $ax + by + cz = d$. Show that the midpoint $(\mathbf{x}_0 + \mathbf{x}_1)/2$ is also a solution.

(b) Continuing from (a), show that every point of the line $(1-t)\mathbf{x}_0 + t\mathbf{x}_1$ is also a solution to the equation $ax + by + cz = d$.

(c) Fill in the blank: If 25 hyperplanes in 12-dimensional space meet at two points $\mathbf{x}_0$ and $\mathbf{x}_1$, then they must also meet at ______ .

Problem 2. Consider the following linear system:

$$\begin{cases} x + y + z = 2 \\ x + 2y + z = 3 \\ x + 3y + 2z = 5 \end{cases}$$

(a) Compute the RREF of the system.

(b) Describe the row picture of the solution.

(c) Describe the column picture of the solution.

Problem 3. Now consider the modified system:

$$\begin{cases} x + y + z = 2 \\ x + 2y + z = 3 \\ 2x + 3y + 2z = c \end{cases}$$

where c is an arbitrary constant.

(a) Put the system in staircase form. You don't need to compute the RREF.

(b) Fill in the blanks: The first two planes meet in a line L. When $c = 5$ we have ∞ many solutions because the third plane ______ , but when $c = 6$ we have 0 solutions because the third plane ______ .

(c) Fill in the blank: It is impossible for the system to have exactly 1 solution because if we have one solution

$$x_1 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + y_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + z_1 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ c \end{pmatrix},$$

then we also have another solution ______ . [Hint: Change x_1 and z_1 somehow. The value of c is irrelevant.]

Problem 4. Consider the following linear system:

$$\begin{cases} 0 + x_2 + 0 + x_4 - x_5 - 4x_6 = -1 \\ x_1 + 2x_2 - x_3 + 4x_4 - x_5 - 4x_6 = 3 \\ x_1 + 2x_2 - x_3 + 4x_4 + 0 - x_6 = 5 \end{cases}$$

(a) Compute the RREF of the system.

(b) Write down the full solution in parametric form.

(c) Describe the row picture of the solution.