Math 210 **Fall 2019**
Homework 1 **Drew Armstrong**

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1. Define the standard basis vectors $\mathbf{e}_1 = (1,0,0)$, $\mathbf{e}_2 = (0,1,0)$ and $\mathbf{e}_3 = (0,0,1)$.

(a) Draw the cube with the following 8 vertices:

$$\mathbf{0},\ \mathbf{e}_1,\ \mathbf{e}_2,\ \mathbf{e}_3,\ \mathbf{e}_1+\mathbf{e}_2,\ \mathbf{e}_1+\mathbf{e}_3,\ \mathbf{e}_2+\mathbf{e}_3,\ \mathbf{e}_1+\mathbf{e}_2+\mathbf{e}_3.$$

(b) Draw the triangle in 3D with corners at $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$. Compute the side lengths and the angles of this triangle by using the dot product.

2. Let $\mathbf{u} = (1,2)$ and $\mathbf{v} = (3,1)$.

(a) Draw the points $\mathbf{u}$ and $\mathbf{v}$ together with the points

$$\frac{1}{2}\mathbf{u}+\frac{1}{2}\mathbf{v},\quad \frac{1}{4}\mathbf{u}+\frac{3}{4}\mathbf{v},\quad \frac{1}{4}\mathbf{u}+\frac{1}{4}\mathbf{v},\quad \mathbf{u}+\mathbf{v}.$$

(b) Draw the infinite line $\{t\mathbf{v}+t\mathbf{u}\}$ where t is any real number. [Hint: It is enough to draw two points on this line and then use a ruler.]

(c) Draw the infinite line $\{t\mathbf{u}+(1-t)\mathbf{v}\}$ where t is any real number. [Hint: Same as (b).]

(d) Shade the finite region of the plane defined by $\{s\mathbf{u}+t\mathbf{v} : 0 \leq s \leq 1, 0 \leq t \leq 1\}$.

(e) Shade the infinite region of the plane defined by $\{s\mathbf{u}+t\mathbf{v} : 0 \leq s, 0 \leq t\}$.

3. Let $\mathbf{u}$ and $\mathbf{v}$ be any two vectors satisfying

$$\mathbf{u}\bullet\mathbf{u} = \mathbf{v}\bullet\mathbf{v} = 1 \qquad \text{and} \qquad \mathbf{u}\bullet\mathbf{v} = 0.$$

Compute the following dot products:

(a) $\mathbf{u}\bullet(-\mathbf{u})$

(b) $(\mathbf{u}+\mathbf{v})\bullet(\mathbf{u}-\mathbf{v})$

(c) $(\mathbf{u}+2\mathbf{v})\bullet(\mathbf{u}-2\mathbf{v})$

4. Lines and Planes

(a) The set of vectors $\mathbf{x} = (x,y)$ that are perpendicular to $\mathbf{v} = (2,1)$ form a line. Draw this line and find its equation.

(b) Draw the line that is parallel to the vector $\mathbf{v} = (2,1)$ and contains the point $(1,3)$. Find the equation of this line.

(c) Describe the set of vectors $\mathbf{x} = (x,y,z)$ that are perpendicular to the vector $(1,1,1)$. Try to draw a picture.

(d) Describe the set of vectors $\mathbf{x} = (x,y,z)$ that are simultaneously perpendicular to each of the vectors $(1,1,1)$ and $(1,2,3)$. Try to draw a picture.

5. Associativity of vector addition. Consider $\mathbf{u} = (u_1,u_2)$, $\mathbf{v} = (v_1,v_2)$ and $\mathbf{w} = (w_1,w_2)$.

(a) Use algebra to prove that $\mathbf{u}+(\mathbf{v}+\mathbf{w}) = (\mathbf{u}+\mathbf{v})+\mathbf{w}$. [Hint: You may assume that addition of numbers is associative.]

(b) Draw a picture to demonstrate that $\mathbf{u}+(\mathbf{v}+\mathbf{w}) = (\mathbf{u}+\mathbf{v})+\mathbf{w}$.