Math 161 **Summer 2015**
Quiz 5 **Drew Armstrong**

Each problem is worth 2 points.

1. Compute the derivative of $f(x) = x^2 + 2^x$.

$$f'(x) = (x^2)' + (2^x)' = 2x + \ln(2) \cdot 2^x.$$

2. Use integration by parts to compute the most general **anti**derivative of $g(x) = x \cdot \ln(x)$.

Let $f(x) = \ln(x)$ and $g'(x) = x$, so that $f'(x) = 1/x$ and $g(x) = x^2/2$. Then we have

$$\begin{aligned}
\int f(x)g'(x)\,dx &= f(x)g(x) - \int f'(x)g(x)\,dx \\
\int x \cdot \ln(x)\,dx &= \frac{1}{2}x^2\ln(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int x\,dx \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\cdot\frac{1}{2}x^2 + C \\
&= \frac{1}{2}x^2\left(\ln(x) - \frac{1}{2}\right) + C.
\end{aligned}$$

3. Compute the derivative of $h(x) = x^x$.

There are two ways to do this.

- First, we could write $h(x) = x^x = \left(e^{\ln(x)}\right)^x = e^{(x\ln(x))}$. Then we use the chain and product rules:

$$\begin{aligned}
h'(x) &= e^{(x\ln(x))} \cdot (x\ln(x))' \\
&= e^{(x\ln(x))}\left(1 \cdot \ln(x) + x \cdot \frac{1}{x}\right) \\
&= x^x(\ln(x) + 1).
\end{aligned}$$

- Second, we could take the natural log of both sides to get $\ln(h) = \ln(x^x) = x\ln(x)$. Then we apply $\frac{d}{dx}$ to both sides to get

$$\begin{aligned}
\frac{d}{dx}\ln(h) &= \frac{d}{dx}(x\ln(x)) \\
\frac{1}{h}\cdot\frac{dh}{dx} &= 1 \cdot \ln(x) + x \cdot \frac{1}{x} \\
\frac{dh}{dx} &= h(\ln(x) + 1) \\
h'(x) &= x^x(\ln(x) + 1).
\end{aligned}$$

4. Use the method of substitution to compute $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx$.

We will use the substitution $u = x^2+1$, so that $du = 2x\,dx$. Then we have

$$\begin{aligned}\int_0^1 \frac{x}{x^2+1}\,dx &= \int_0^1 \frac{1}{u}(x\,dx)\\ &= \int_1^2 \frac{1}{u}\left(\frac{du}{2}\right)\\ &= \frac{1}{2}\int_1^2 \frac{1}{u}\,du\\ &= \frac{1}{2}\ln|u|\Big|_1^2\\ &= \frac{1}{2}(\ln(2)-\ln(1)).\end{aligned}$$

5. When $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, the function arcsin is defined by

$$\boxed{y = \arcsin(x) \quad\Longleftrightarrow\quad x = \sin(y).}$$

Apply $\frac{d}{dx}$ to both sides of the equation $x = \sin(y)$ and then use this to compute $\frac{dy}{dx}$. You do not need to simplify your answer.

First we have

$$\begin{aligned}\frac{d}{dx}x &= \frac{d}{dx}\sin(y)\\ 1 &= \cos(y)\cdot\frac{dy}{dx}.\end{aligned}$$

Then solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\cos(\arcsin(x))}.$$

We weren't asked to simplify this, but if we remember that $\cos(\arcsin(x)) = \sqrt{1-x^2}$ we can write

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}.$$