Math 161 **Summer 2015**
Quiz 3 **Drew Armstrong**

Problems **1** and **2** refer to the function $f(x) = \frac{1}{3}x^3 - x$.

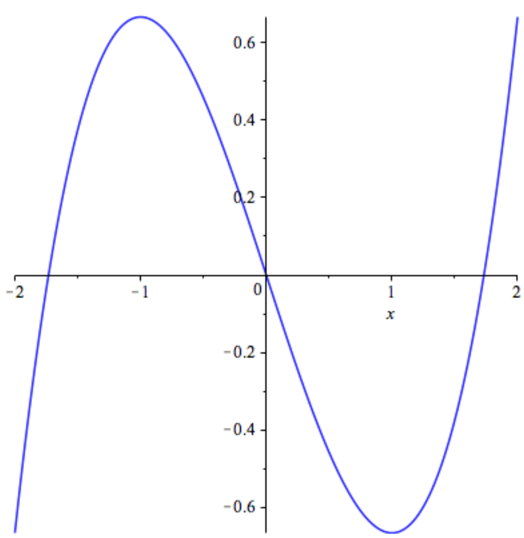
1. [3 points] Compute $f'(x)$ and show that $f'(x) = 0$ when $x = -1$ or $x = +1$. Determine when $f(x)$ is increasing or decreasing.

We have
$$f'(x) = \frac{1}{3}3x^2 - 1 = x^2 - 1 = (x+1)(x-1)$$
and we conclude that $f'(x) = (x+1)(x-1) = 0$ when $(x+1) = 0$ (i.e., $x = -1$) or $(x-1) = 0$ (i.e., $x = +1$). When $\mathbf{x < -1}$ we have $(x+1) < 0$ and $(x-1) < 0$, hence $f'(x) = (x+1)(x-1) > 0$, i.e., $f(x)$ is **increasing**. When $\mathbf{-1 < x < 1}$ we have $(x+1) > 0$ and $(x-1) < 0$, hence $f'(x) = (x+1)(x-1) < 0$, i.e., $f(x)$ is **decreasing**. Finally, when $\mathbf{1 < x}$ we have $(x+1) > 0$ and $(x-1) > 0$, hence $f'(x) = (x+1)(x-1) > 0$, i.e., $f(x)$ is **increasing**.

2. [3 points] Compute $f''(x)$ and determine when the graph of $f(x)$ is concave up or concave down. Determine whether $x = -1$ and $x = +1$ are maxima or minima (or neither).

First we compute
$$f''(x) = (x^2 - 1)' = 2x.$$
Note that $f''(x) = 0$ when $x = 0$ (here f has an inflection), $f''(x) < 0$ when $x < 0$ (here f is concave down) and $f''(x) > 0$ when $x > 0$ (here f is concave up). In particular, we have $f''(-1) < 0$ and $f''(+1) > 0$, so $x = -1$ is a maximum and $x = +1$ is a minimum.

I didn't ask you to draw it, but here is the graph of $f(x)$ for refence:



For problems **3** and **4**, suppose that x and y are quantities related by $y = \dfrac{1}{+\sqrt{3+x^2}}$.

3. [2 points] Compute $\dfrac{dy}{dx}$.

First we write $y = (3+x^2)^{-1/2}$. Then we use the chain rule to compute

$$\frac{dy}{dx} = -\frac{1}{2}(3+x^2)^{-3/2}(3+x^2)' = \frac{-1}{2(3+x^2)^{3/2}}(2x) = \frac{-x}{(3+x^2)^{3/2}}.$$

4. [2 points] Suppose we measure x to find $x = 1$ unit with a possible error of $dx = 0.1$ units. Compute the value of y and use your answer from Problem **3** to estimate the possible error in y.

When $x = 1$ the value of y is $\dfrac{1}{+\sqrt{3+1^2}} = \dfrac{1}{+\sqrt{4}} = \dfrac{1}{2}$.

From Problem **3** we have

$$\begin{aligned} dy &= \frac{-x}{(3+x^2)^{3/2}}\,dx \\ &= \frac{-1}{(3+1^2)^{3/2}}\,(0.1) \\ &= \frac{-1}{4^{3/2}}(0.1) \\ &= \frac{-1}{8}(0.1) \\ &= -0.125. \end{aligned}$$

Thus, if $x = 1 \pm 0.1$ we conclude that $y = 0.5 \pm 0.125$.