Math 161 **Summer 2015**
Quiz 1 **Drew Armstrong**

Compute the following limits, or say why they do not exist.

1. $\lim_{x\to 1} \frac{x^2-4}{x-2}$

Here we can just plug in $x = 1$ to get

$$\lim_{x\to 1} \frac{x^2-4}{x-2} = \frac{1^2-4}{1-2} = \frac{-3}{-1} = 3.$$

2. $\lim_{x\to 2} \frac{x^2-4}{x-2}$

Here the limit has indeterminate form "0/0" so we need a trick. We factor the numerator to obtain

$$\begin{aligned}\lim_{x\to 2} \frac{x^2-4}{x-2} &= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)} \\ &= \lim_{x\to 2}(x+2) \\ &= 2+2 \\ &= 4.\end{aligned}$$

3. $\lim_{n\to\infty} \frac{1+1/n}{\cos(\pi/n)}$

First note that $\lim_{n\to\infty} \cos(\pi/n) = \cos\left(\lim_{n\to\infty} \pi/n\right) = \cos(0) = 1$ and $\lim_{n\to\infty} 1/n = 0$. Thus we have

$$\lim_{n\to\infty} \frac{1+1/n}{\cos(\pi/n)} = \frac{1+\lim_{n\to\infty} 1/n}{\lim_{n\to\infty} \cos(\pi/n)} = \frac{1+0}{1} = 1.$$

4. $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

Here the limit has indeterminate form "$\infty - \infty$" so we need a trick. First we find a common denominator and then simplify to obtain

$$
\begin{aligned}
\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) &= \lim_{t\to 0}\left(\frac{t^2+t}{t(t^2+t)}-\frac{t}{t(t^2+t)}\right)\\
&= \lim_{t\to 0}\frac{(t^2+t)-t}{t(t^2+t)}\\
&= \lim_{t\to 0}\frac{t^2}{t^2(t+1)}\\
&= \lim_{t\to 0}\frac{1}{t+1}\\
&= \lim_{t\to 0}\frac{1}{0+1}\\
&= 1.
\end{aligned}
$$

5. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+4x+1}-x\right)$

Here the limit has indeterminate form "$\infty-\infty$" so we need a trick. We multiply and divide by the conjugate expression to obtain

$$
\begin{aligned}
\lim_{x\to\infty}\left(\sqrt{x^2+4x+1}-x\right) &= \lim_{x\to\infty}\left(\sqrt{x^2+4x+1}-x\right)\frac{\sqrt{x^2+4x+1}+x}{\sqrt{x^2+4x+1}+x}\\
&= \lim_{x\to\infty}\frac{(x^2+4x+1)-x^2}{\sqrt{x^2+4x+1}+x}\\
&= \lim_{x\to\infty}\frac{4x+1}{\sqrt{x^2+4x+1}+x}.
\end{aligned}
$$

Now we have a limit of indeterminate form "∞/∞" so we need another trick. We multiply the numerator and denominator both by $1/x$ to obtain

$$
\begin{aligned}
\lim_{x\to\infty}\frac{4x+1}{\sqrt{x^2+4x+1}+x} &= \lim_{x\to\infty}\frac{4+1/x}{(1/x)\sqrt{x^2+4x+1}+1}\\
&= \lim_{x\to\infty}\frac{4+1/x}{\sqrt{1/x^2}\sqrt{x^2+4x+1}+1}\\
&= \lim_{x\to\infty}\frac{4+1/x}{\sqrt{1+4/x+1/x^2}+1}\\
&= \frac{4+0}{\sqrt{1+0+0}+1}\\
&= 2.
\end{aligned}
$$