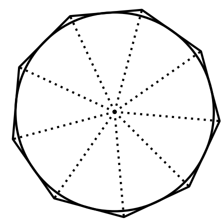
Math 161 **Summer 2015**
Homework 1 **Drew Armstrong**

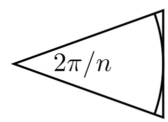
---

1. Let P_n be a regular polygon with n sides and let C be the largest circle contained inside P_n. Suppose that C has radius r.



(a) Compute an exact formula for the **perimeter** of P_n.
(b) Compute an exact formula for the **area** of P_n.

[Hint: Divide the polygon into n triangles at its center and consider one of the triangles.



Use the fact that the angle at the center is $2\pi/n$ radians.]

2. (a) Use a calculator to compute the value of $n\,\tan(\pi/n)$ for $n = 1, 10, 100, 1000, 10000$. Now guess the exact value of the limit

$$\lim_{n\to\infty} n\,\tan(\pi/n).$$

(b) Explain how your guess in part (a) agrees with your solution to Problem 1. [Hint: The limit of the perimeter of P_n as n approaches ∞ **should** be the circumference of the circle, i.e., $2\pi r$.]

3. We showed in class that the region between the graph of $f(x) = x^2$ and the x-axis, from $x = 0$ to $x = 1$, is exactly $1/3$. In this problem you will show that the area between the graph of $g(x) = x^3$ and the x-axis, from $x = 0$ to $x = 1$, is exactly $1/4$.

(a) Draw a picture of this region.
(b) Divide the interval between $x = 0$ and $x = 1$ into n equal intervals of width $1/n$. On the interval from $x = (i-1)/n$ to $x = i/n$ draw a rectangle of height $(i/n)^3$. Write out an expression for the total area of these n rectangles.
(c) Compute the limit of your expression from part (b) as n approaches ∞. [Hint: You should use the algebraic formula

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2.$$

You do not need to say why this mysterious formula is true.]