MTH 309 Homework

1. (Due: Jan. 26, 2006) 1.1 / 1, 3, 5, 8, 10, 13, 14, 15, 17, 19, 20, 23, 24, 25, 27, 34,
   Additional: Use Venn diagrams to illustrate distributive properties and De-Morgan's Laws.

2. (Due: Feb. 2, 2006) 1.4 / 1-7, 9, 11, 15-19, 21, 23, 24

3. (Due: Feb. 9, 2006)
   - 1.4/ 26, 27, 30-32, 34
   - 1.5 / 1-3, 8-10, 12, 17, 18
   - Additional: Find the inverse of the following functions:
     (a) Let \( \Sigma = \{0, 1\} \) and
     \[
     f : \Sigma_n \rightarrow \{w \in \Sigma_{n+1} : \text{last letter of } w \text{ is } 1\}
     \]
     \[
     f(w_1 \ldots w_n) = w_1 \ldots w_n 1
     \]
     (b) Let \( S = \{1, \ldots, n\} \) and
     \[
     f : \mathcal{P}(S) \rightarrow \{T \in \mathcal{P}(S \cup \{n + 1\}) : n + 1 \in T\}
     \]
     \[
     f(U) = U \cup \{n + 1\}
     \]

4. (Due: Feb. 16, 2006)
   - 1.5 / 26, 41
   - Additional problems:
     (a) Use mathematical induction to prove that \( 2^n > n^2 \) for all \( n \geq 5 \).
     (b) Let \( a_0, a_1, \ldots \) be the sequence that is defined recursively by
     \[
     a_0 = 1, \quad a_1 = 3
     \]
     \[
     a_n = 5a_{n-1} - 6a_{n-2} \quad \forall n \geq 2.
     \]
     Use mathematical induction to prove \( a_n = 3^n \) for all \( n \geq 0 \).
     (c) Use mathematical induction to prove that postage of 24 cents or more can be achieved by using only 5 cent and 7 cent stamps.
     (d) Use mathematical induction to prove the following proposition: Every integer greater than 2 either has a prime divisor or is divisible by 4.
(e) Use “proof by contradiction” to show that for all integers $n$, if $n^2$ is even then so is $n$.

(f) Use “proof by contradiction” to show that $\sqrt{3}$ is irrational.

5. (Due: Feb. 23, 2006)
   - 2.1/ 1, 2, 3, 5abcf, 6ab, 8abdef
   - 2.2/ 1-4, 9, 10, 12, 17

6. (Due: March 2, 2006) 2.2 / 18, 19, 20

7. Exam: March 2, 2006

8. (Due: March 9, 2006)
   - 2.3 / 4,5, 14, 17cef, 22, 23
   - Additional problems:
     (1) Prove Properties 4 and 5 of sets given on page 13 by using the corresponding property of logic given on page 82.
     (2) Let $A, B, C, D$ be sets. Use properties of logic to prove 
        \[(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)\]
     (3) Translate the following quantified statements into symbolic form. Be sure to indicate what the statement variables stand for.
        (a) Every student in this class is a high school graduate.
        (b) There is a student in this class who owns a computer.
        (c) For every integer there is a prime that is bigger than that integer.
        (d) There is a prime that is bigger than every integer.
        (e) Some cats are pets, and all cats are neither dogs nor rabbits.
     (4) Give negations of the statements in (3) in symbolic form (start with a quantifier) and also express negations in English.
     (5) Determine the truth values of (3c), (3d) and (3e).
     (6) Determine the truth values of
        (a) $\exists x \in \mathbb{Z}, x^2 = x$
        (b) $\forall x \in \mathbb{Z}, x^2 = x$
        (c) $\exists x \in \mathbb{Z}, x = x + 1$
        (d) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 = y$
        (e) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x = y^2$
(e) \[ \exists y \in \mathbb{R}, \forall x \in \mathbb{R}, x^2 = y \]

9. (Due: March 23)
   - 4.2 / 1, 4, 6, 7, 9, 10, 12, 13, 15, 16, 17, 20, 21
   - 4.4 / 1, 2, 6, 7, 8, 9, 10, 12, 16, 24, 26

10. (Due: March 30)
    - 4.5 / 1, 2, 3, 4, 6, 8, 9, 10, 16
    - 4.6 / 1, 2, 3, 6, 8, 9, 11, 13, 14, 17, 18, 27, 29, 30
    - Additional problem: Let \( k \leq n \). Find a bijection from the set of subsets of \( \{1,2,\ldots,n\} \) of size \( k \) to the set of subsets of \( \{1,2,\ldots,n\} \) of size \( n - k \). This proves \( C(n, k) = C(n, n - k) \).

11. (Due: April 6)
    - 4.7 / 1, 4, 5, 6, 7, 8, 10, 11, 13, 16, 17, 20, 21
    - Extra credit problem: Give a bijection which proves that for all \( n \), the number of subsets of \( \{1,2,\ldots,n\} \) of even cardinality equals the number of subsets of \( \{1,2,\ldots,n\} \) of odd cardinality.

12. (Due: April 13)
    - 4.8 / 1, 2, 4, 5, 6, 8, 9, 10, 11, 13, 15, 17, 20
    - Additional problems:
      1. Find the number of partitions of \( \{1,2,\ldots,35\} \) into 7 blocks of equal size.
      2. Find the number of partitions of \( \{1,2,\ldots,43\} \) into 3 blocks of size 5 and 4 blocks of size 7.
      3. Find the number of partitions of \( \{1,2,\ldots,18\} \) into 1 block of size 3, 1 block of size 4, 1 block of size 5, and 1 block of size 6.
    - Extra credit problem: Fix positive integers \( a_1, a_2, \ldots, a_m \) and let
      \[ n = 1a_1 + 2a_2 + \ldots + ma_m. \]
      Find the number of partitions of \( \{1,2,\ldots,n\} \) into \( a_i \) blocks of size \( i \) for each \( i = 1,2,\ldots,m \).

13.  
    - 4.9 / 1, 2, 7, 10, 12, 21, 22, 24, 25
    - 4.10 / 1-8, 18