The exam will consist of **about nine** questions (which may be further divided into **several parts**), with **unequal** weights. You are expected to know how to do the following types of problems (this list is by no means exhaustive, I just want to give you an idea of what you should expect in the exam. For further practice, do the recommended questions or the problems in the book):

1. **Inverse functions, log, exponential, trigonometric functions, and their calculus**

   - Find \((f^{-1})'(a)\). \(f(x) = x^3 + 3 \sin x + 2 \cos x, \ a = 2\)
   - Use the Laws of Logarithms to expand the quantity.

   \[
   \ln \sqrt[3]{\frac{x - 1}{x + 1}}
   \]

   - Express the quantity as a single logarithm.

   \[
   \ln 3 + \frac{1}{3} \ln 8
   \]

   - Use the Laws of Logarithms to expand the quantity.

   \[
   \ln s^4 \sqrt{t \sqrt{u}}
   \]

   - Differentiate the function.

   \[
   y = \ln |2 - x - 5x^2|
   \]

   \[
   g(x) = \ln(x\sqrt{x^2 - 1})
   \]

   - Use logarithmic differentiation to find the derivative of the function.

   \[
   y = \sqrt{\frac{x - 1}{x^4 + 1}}
   \]
Evaluate the integral.
\[ \int_1^2 \frac{dt}{8 - 3t} \]
\[ \int_4^9 \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 \, dx \]
\[ \int_e^6 \frac{dx}{x \ln x} \]
\[ \int_1^e \frac{x^2 + x + 1}{x} \, dx \]

Differentiate the function.
\[ y = \frac{e^u - e^{-u}}{e^u + e^{-u}} \]
\[ y = x^{\cos x} \]
\[ y = \sqrt{x^x} \]

Evaluate the integral.
\[ \int_0^1 \frac{\sqrt{1 + e^{-x}}}{e^x} \, dx \quad \int \frac{2^x}{2^x + 1} \, dx \]

Find the exact value of each expression.
\[ \sin^{-1}(\sqrt{3}/2) \]
\[ \tan^{-1}(1/\sqrt{3}) \]
\[ \text{arctan} \ 1 \]
• Find the derivative of the function.
  \[ y = \tan^{-1}(x^2) \]
  \[ y = \sin^{-1}(2x + 1) \]

• Evaluate the integral.
  \[ \int_{0}^{\sqrt{\frac{3}{4}}} \frac{dx}{1 + 16x^2} \]
  \[ \int \frac{t^2}{\sqrt{1 - t^6}} \, dt \]
  \[ \int_{0}^{\frac{1}{2}} \frac{\sin^{-1}x}{\sqrt{1 - x^2}} \, dx \]

• Find the derivative.
  \[ \cosh(\ln x) \]
  \[ \sinh^{-1}(\tan x) \]

• Evaluate the integral.
  \[ \int \frac{\cosh x}{\cosh^2 x - 1} \, dx \]
  \[ \int_{4}^{6} \frac{1}{\sqrt{t^2 - 9}} \, dt \]
  \[ \int \frac{e^x}{1 - e^{2x}} \, dx \]

2. L’Hospital Rule
3. Integration by parts, Trigonometric integration/substitution

- Evaluate the integral.
  \[ \int \sin^{-1} x \, dx \]
  \[ \int \cos \sqrt{x} \, dx \]
  \[ \int_{1}^{4} e^\sqrt{x} \, dx \]
  \[ \int \sin^2 x \cos^3 x \, dx \]

- Evaluate the integral using trigonometric substitution.
  \[ \int \frac{dx}{x^2\sqrt{4 - x^2}} \]
  \[ \int \frac{x}{\sqrt{1 + x^2}} \, dx \]

(You may need: \((\sec x)' = \sec x \tan x\))
4. Partial Fractions

- Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

\[
\int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6} \, dx
\]

Evaluate the integral.

\[
\int_0^1 \frac{t^6 + 1}{t^6 + t^3} \, dt
\]

\[
\int_0^1 \frac{t^6 + 1}{(x^2 - x)(x^4 + 2x^2 + 1)} \, dx
\]

5. Improper integrals

- Use the Comparison Theorem to determine whether the integral is convergent or divergent.

\[
\int_0^\infty \frac{x}{x^3 + 1} \, dx
\]

\[
\int_1^\infty \frac{2 + e^{-x}}{x} \, dx
\]

- Determine whether each integral is convergent or divergent. Evaluate those that are convergent.
6. Power Series

- Find a power series representation for the function
  
  \[ f(x) = \frac{1}{x + 10} \]

- Find the Taylor Series of \( f \) centered at \( x = 0 \):
  
  \[ f(x) = \sinh x \]

- Find the Taylor Series of \( f \) centered at \( x = a \):
  
  \[ f(x) = \cos x, \quad a = \pi \]

  Evaluate the indefinite integral as an infinite series.

  \[ \int \frac{e^x - 1}{x} \, dx \]

- Use the first four non-zero terms of the Taylor series to find an approximate value of the integral:

  \[ \int_0^1 x \cos(x^3) \, dx \]
7. Polar coordinates, parametric curves

- Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter. 
  \( x = \sin^3 \theta, \quad y = \cos^3 \theta; \quad \theta = \pi/6 \)

- Find the area enclosed by the x-axis and the curve 
  \( x = 1 + e^t, \quad y = t - t^2. \)

- Find the exact length of the curve. 
  \( x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 1 \)

- Find \( dy/dx. \)
  \( x = t \sin t, \quad y = t^2 + t \)

- Identify the curve by finding a Cartesian equation for the curve. 
  \( r = 2 \cos \theta \)

- Find a polar equation for the curve represented by the given Cartesian equation. 
  \( y = 1 + 3x \)

- Find the slope of the tangent line to the given polar curve at the point specified by the value of \( \theta. \)
  \( r = 2 - \sin \theta, \quad \theta = \pi/3 \)
Find the area of the region enclosed by one loop of
the curve.
\[ r = 4 \cos 3\theta \]
\[ r^2 = \sin 2\theta \]
Find the exact length of the polar curve.
\[ r = 3 \sin \theta, \quad 0 \leq \theta \leq \pi/3 \]
\[ r = e^{2\theta}, \quad 0 \leq \theta \leq 2\pi \]