MTH 162 Homework 1

Do the first four problems. Due: Jan 22, 2014 (Wednesday). Hand in to me during the class.

Compulsory:

Ex 5.1

21-26  ▪ Find a formula for the inverse of the function.
23. \( f(x) = 1 + \sqrt{2 + 3x} \)

37-40  ▪ Find \((f^{-1})'(a)\).
38. \( f(x) = x^3 + 3 \sin x + 2 \cos x, \quad a = 2 \)

Ex 5.2

1-4  ▪ Use the Laws of Logarithms to expand the quantity.
2. \( \ln \sqrt[3]{\frac{x-1}{x+1}} \)

5-8  ▪ Express the quantity as a single logarithm.
6. \( \ln 3 + \frac{1}{3} \ln 8 \)  
   (Please fully simplify your answer)
**Recommended:** (These types of questions may also appear in the exams)

**Ex 5.1**

17. If \( h(x) = x + \sqrt{x} \), find \( h^{-1}(6) \).

21–26 □ Find a formula for the inverse of the function.

22. \( f(x) = \frac{4x - 1}{2x + 3} \)

24. \( y = 2x^3 + 3 \)

25. \( y = \frac{1 - \sqrt{x}}{1 + \sqrt{x}} \)

26. \( f(x) = 2x^2 - 8x, \quad x \geq 2 \)

37–40 □ Find \((f^{-1})'(a)\).

37. \( f(x) = 2x^3 + 3x^2 + 7x + 4, \quad a = 4 \)

39. \( f(x) = 3 + x^2 + \tan(\pi x/2), \quad -1 < x < 1, \quad a = 3 \)

40. \( f(x) = \sqrt{x^3 + x^2 + x + 1}, \quad a = 2 \)

**Ex. 5.2**

1–4 □ Use the Laws of Logarithms to expand the quantity.

1. \( \ln \sqrt{ab} \)

4. \( \ln s^4 \sqrt{t \sqrt{u}} \)

5–8 □ Express the quantity as a single logarithm.

5. \( \ln 5 + 5 \ln 3 \)

7. \( \frac{1}{3} \ln(x + 2)^3 + \frac{1}{2} \ln [\ln x - \ln(x^2 + 3x + 2)^2] \)

8. \( \ln(a + b) + \ln(a - b) - 2 \ln c \)
Challenging: (Harder problems. Attempt if you are interested.)

Ex. 5.1

31. Let \( f(x) = \sqrt{1 - x^2}, \ 0 \leq x \leq 1. \)
   (a) Find \( f^{-1}. \) How is it related to \( f? \)
   (b) Identify the graph of \( f \) and explain your answer to
       part (a).

43. If \( f(x) = \int_{3}^{x} \sqrt{1 + t^3} \, dt, \) find \( (f^{-1})'(0). \)
   (Hint: what’s \( a = f^{1}(0)? \) Use the 2\(^{nd}\)
   fundamental theorem of calculus. )

48. (a) If \( f \) is a one-to-one, twice differentiable function with
     inverse function \( g, \) show that
     \[
     g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}
     \]
     (b) Deduce that if \( f \) is increasing and concave upward, then
         its inverse function is concave downward.

Ex 5.2

69. By comparing areas, show that
\[
\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < \ln n < 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}
\]

73. Use the definition of derivative to prove that
\[
\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1
\]
   (Hint: \( \ln 1 = 0, \) how’s
   related to the derivative of \( \ln? \) )