1.5 - 16
4 Lose, 1 Win chips

\[ P(\text{the first player wins}) = P(\text{the white chip is drawn on an odd draw}) \]

\[ = \sum_{x=1, 2, \ldots, \text{odd}} f(x) \]

\[ f(x) = \frac{1}{5} \quad \forall \text{ } x = 1, 2, 3, 4, 5 \]

there are many ways to see that. Directly, it is

\[ f(x) = P(\text{ } A \cap B ) = P(\text{A | B}) P(\text{B}) \]

\[ A = \{ \text{the } x^{th} \text{ chip is } W \} \]

\[ B = \{ \text{there are exactly zero } W \text{ among the first } x-1 \text{ draws} \} \]

\[ f(x) = \frac{1}{5-x+1} \cdot \frac{1}{\binom{5}{x-1}} = \frac{1}{5} \text{ (after the algebra)} \]

So \[ P(\text{1st player wins}) = f(1) + f(3) + f(5) = \frac{3}{5} \]
Problem 1.5-16 continued

with replacement \( x = 1, 2, 3, \ldots \) any number is possible

\[
P(A \cap B) = P(A \mid B) \cdot P(B)
\]

\[
\frac{1}{5} \cdot P\left( \text{there are exactly zero } W \text{ among the first } x-1 \text{ draws} \right)
\]

\[
\left( \begin{array}{c} x-1 \\ 0 \end{array} \right) \left( \frac{1}{5} \right)^0 \left( \frac{4}{5} \right)^{x-1}
\]

\[
f(x) = \frac{1}{5} \cdot \left( \frac{4}{5} \right)^{x-1}
\]

denote \( p = \frac{1}{5} \).

\[
f(1) + f(3) + f(5) + \ldots
\]

\[
= \frac{1}{5} \left[ \left( \frac{4}{5} \right)^{1-1} + \left( \frac{4}{5} \right)^{3-1} + \left( \frac{4}{5} \right)^{5-1} + \left( \frac{4}{5} \right)^{7-1} + \ldots \right]
\]

\[
= p \left[ 1 + (1-p)^2 + (1-p)^2 \cdot 2 + (1-p)^2 \cdot 3 + \ldots \right]
\]

\[
= p \left[ \frac{1}{1-(1-p)^2} \right] = \frac{p}{(1-1+p)(1+1-p)} = \frac{1}{2-p} = \frac{5}{9}
\]

We used \( 1 + z + z^2 + \ldots = \frac{1}{1-z} \) when \( |z| < 1 \)