Section 4.3: Quadratic Functions and Their Properties

- **Def:** A quadratic function is a function of the form \( f(x) = ax^2 + bx + c \), where \( a, b, c \) are real numbers and \( a \neq 0 \).

- The domain of a quadratic function is all real numbers. The shape of the graph of a quadratic function is called a parabola.

- Every quadratic function \( f(x) = ax^2 + bx + c \) can be written as \( f(x) = a(x - h)^2 + k \), where \( h = -\frac{b}{2a} \) and \( k = \frac{4ac - b^2}{4a} \). So every quadratic function is just like the function \( f(x) = x^2 \), but transformed.

- The graph of the function \( f(x) = x^2 \) opens up and has a lowest point. The graph of the function \( f(x) = -x^2 \) opens down and has a highest point. In general, we call the highest or lowest point of the parabola the vertex.

- **Def:** The axis of symmetry is the vertical line through the vertex, around which the graph of the function is symmetric.

- For the quadratic function \( f(x) = ax^2 + bx + c \), the vertex is always \( \left( -\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right) \) and the axis of symmetry is \( x = -\frac{b}{2a} \). If \( a > 0 \) then the graph opens up and the vertex is the lowest point (minimum). If \( a < 0 \) then the graph opens down and the vertex is the highest point (maximum).

- Recall that to find the \( x \)-intercepts of a function, you set \( y = 0 \) and solve for \( x \). So trying to find the \( x \)-intercepts of the quadratic function \( f(x) = ax^2 + bx + c \) is the same as solving the equation \( ax^2 + bx + c = 0 \), which you can always solve by using the quadratic formula.

- The number of \( x \)-intercepts of a quadratic function depends on whether the graph opens up or down and it also depends on whether the vertex is above or below the \( x \)-axis.

1. If the graph of a quadratic function opens up and the vertex is below the \( x \)-axis or if the graph opens down and the vertex is above the \( x \)-axis, then there will be two \( x \)-intercepts.
2. If the vertex is touching the $x$-axis, then there is one x-intercept regardless of whether the graph opens up or down.

3. If the graph of a quadratic function opens up and the vertex is above the $x$-axis or if the graph opens down and the vertex is below the $x$-axis, then there will be no x-intercepts.

- The range of the quadratic function $f(x) = ax^2 + bx + c = a(x - h)^2 + k$ is:
  1. $[k, \infty)$ if $a > 0$ (i.e., if the graph opens up).
  2. $(-\infty, k]$ if $a < 0$ (i.e., if the graph opens down).

- ex. Write the equation of the quadratic function shown in the graph in the form $f(x) = ax^2 + bx + c$: 
• ex. For the given quadratic function $f$ answer the following:
  i) Does the graph of $f$ open up or down?,
  ii) What is vertex $(h, k)$ of $f$?,
  iii) What are the intercepts of $f$?,
  iv) What is the domain of $f$?,
  v) What is the range of $f$?,
  vi) What are the intervals of increase and decrease of $f$?,
  vii) What does the graph of $f$ look like?

  a) $f(x) = x^2 - 4x$
     i) 

     ii) 

     iii) 

     iv) 

     v) 

     vi) 

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b) \( f(x) = -3x^2 + 3x - 2 \)

i)