01/25

Wednesday, January 25, 2017 5:00 PM

$C(x) = \text{Fixed cost} + \text{Variable cost}$

$R(x) \approx \text{revenue}$

$P(x) \approx \text{profit}$

$P(x) = R(x) - C(x)$

**APPLIED EXAMPLE 3** Profit Functions Refer to Example 2. Suppose the total revenue realized by Puritron from the sale of $x$ water filters is given by the total revenue function

$$R(x) = -0.0005x^2 + 20x \qquad (0 \le x \le 40{,}000)$$

**a.** Find the total profit function—that is, the function that describes the total profit Puritron realizes in manufacturing and selling $x$ water filters per month.

**b.** What is the profit when the level of production is 10,000 filters per month?

$\to C(x) = -0.0001x^2 + 10x + 10000$

$P(x) = -0.0005x^2 + 20x - (-0.0001x^2 + 10x + 10000)$

$= -0.0005x^2 + 20x + 0.0001x^2 - 10x - 10000$

$= \boxed{-0.0004x^2 + 10x - 10000}$

b) $P(10000) = -0.0004(10000)^2 + 10 \cdot 10000 - 10000$

$= \boxed{50000}$

2.3

**APPLIED EXAMPLE 1** Erosion of the Middle Class The idea of a large, stable middle class (defined as those with annual household incomes in 2010 between \$39,000 and \$118,000 for a family of three) is central to America's sense of itself. The following table gives the percentage of middle-income adults in the United States from 1971 through 2011:

| Year | 1971 | 1981 | 1991 | 2001 | 2011 |
|---|---|---|---|---|---|
| **Percent, $y$** | 61 | 59 | 56 | 54 | 51 |

A mathematical model giving the percentage of middle-class adults in the United States for the period under consideration is given by

$$f(t) = -2.5t + 61.2 \qquad (0 \le t \le 4)$$

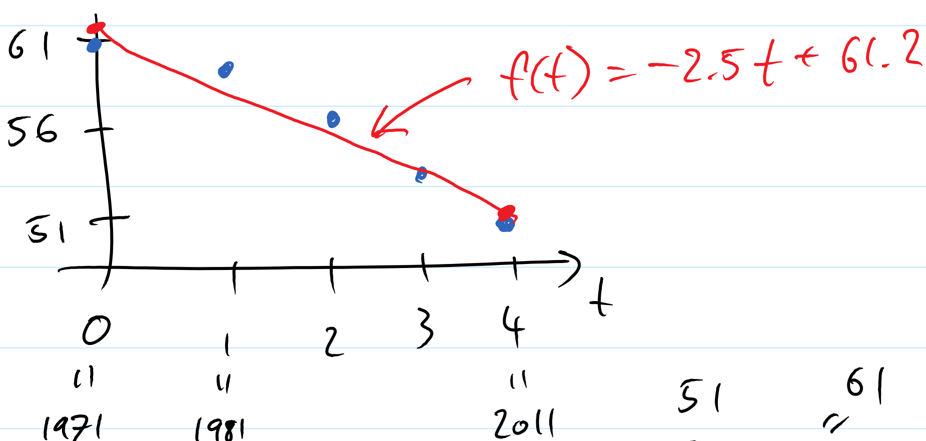
where $t$ is measured in decades, with $t = 0$ corresponding to 1971.

**a.** Plot the data points, and sketch the graph of the function $f$ on the same set of axes.

**b.** What is the rate of change of the percentage of middle-income adults in the United States over the period from 1971 through 2011?

**c.** Assuming that the trend continues, what will the percentage of middle-income adults in the United States be in 2021?

*Source:* Pew Research Center.




b) average rate of change $= \dfrac{\overset{51}{g(4)} - \overset{61}{g(0)}}{4-0} = \dfrac{y(b)-y(a)}{b-a}$

$= \dfrac{51-61}{4} = \dfrac{-10}{4} = \dfrac{-5}{2} = -2.5$

If $f(x) = ax+b$ (a linear function) then the rate of change is $a$.

$f(t) = \underset{\text{rate of change}}{\underbrace{-2.5}}\,t + 61.2$

rate of change

c) $f(5) = -2.5(5) + 61.2 = 61.2 - 12.5 = \boxed{48.7}$

$5 \approx 2021$

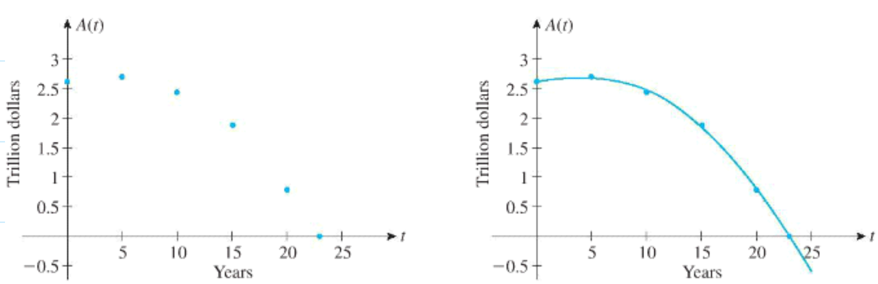
**APPLIED EXAMPLE 3** Social Security Trust Fund Assets The projected assets of the Social Security trust fund (in trillions of dollars) from 2010 through 2033 are given in the following table.

| Year | 2010 | 2015 | 2020 | 2025 | 2030 | 2033 |
|---|---|---|---|---|---|---|
| Assets | 2.61 | 2.68 | 2.44 | 1.87 | 0.78 | 0 |

The scatter plot associated with these data are shown in Figure 19a, where $t = 0$ corresponds to 2010. A mathematical model giving the approximate value of the assets in the trust fund $A(t)$, in trillions of dollars, in year $t$ is

$$A(t) = 0.000008140t^4 - 0.00043833t^3 - 0.0001305t^2 + 0.02202t + 2.612 \quad (0 \le t \le 23)$$

The graph of $A(t)$ is shown in Figure 19b. (You will be asked to construct this model in Exercise 20, Using Technology Exercises 2.3.)




(a) Scatter plot (b) The graph of $A$ together with the scatter plot

FIGURE 19

**a.** The first baby boomers turned 65 in 2011. What were the assets of the Social Security system trust fund at that time? The last of the baby boomers will turn 65 in 2029. What will the assets of the trust fund be at that time?

**b.** Unless payroll taxes are increased significantly and/or benefits are scaled back dramatically, it is a matter of time before the assets of the current system are depleted. Use the graph of the function $A(t)$ to estimate the year in which the current Social Security system is projected to go broke.

*Source*: Social Security Administration.

a) Since $2010 \rightarrow t=0$ then $2011 \rightarrow t=1$

$A(1) = 0.000008140 \cdot 1^4 - 0.00043833 \cdot 1^3 - 0.0001305 \cdot 1^2$

$A(1) = 0.000008140 \cdot 1^4 - 0.00043833 \cdot 1^3 - 0.0001305 \cdot 1^2$
$+ 0.02202 \cdot 1 + 2.612 = 2.633$

$2029 \rightarrow t = \boxed{19}$

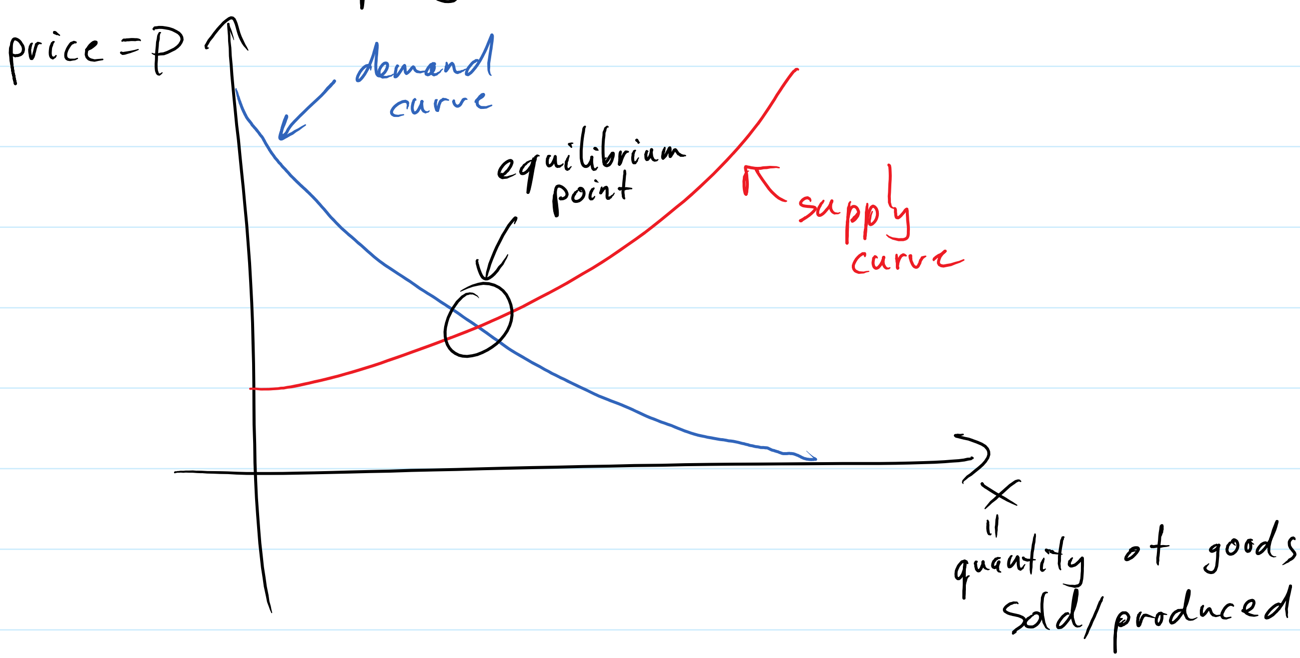
$2010 + t = 2029$

$t = 2029 - 2010 = 19$

$A(19) \simeq 1.038$

b) $t = 23 \rightarrow$ year $= \boxed{2033}$

Demand / Supply curves



**APPLIED EXAMPLE 5** Supply-Demand for Bluetooth Headsets The demand function for a certain brand of Bluetooth wireless headsets is given by

$$p = d(x) = -0.025x^2 - 0.5x + 60$$

and the corresponding supply function is given by

$$p = s(x) = 0.02x^2 + 0.6x + 20$$

where $p$ is expressed in dollars and $x$ is measured in units of a thousand. Find the equilibrium quantity and price.

Set $d(x) = s(x)$

$$-0.025x^2 - 0.5x + 60 = 0.02x^2 + 0.6x + 20$$

$$\frac{-0.045x^2 - 1.1x + 40}{-0.045} = \frac{0}{-0.045}$$

$$x^2 + 24.4x - 888.8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-24.4 \pm \sqrt{24.4^2 - 4 \cdot 1 \cdot (-888.8)}}{2}$$

$$x = \frac{-24.4 \pm 64.42}{2}$$

20

$-\cancel{44}$

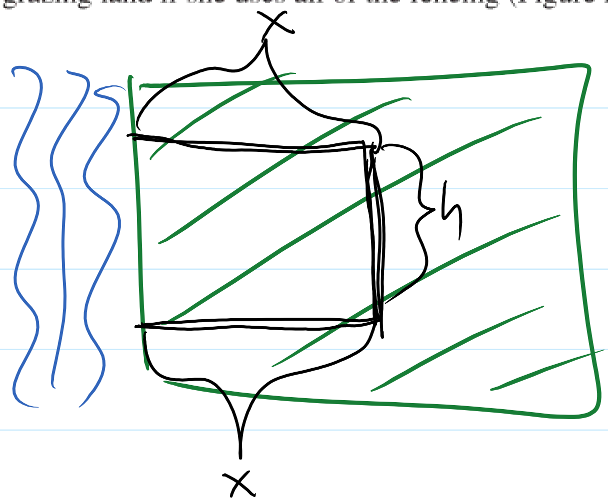
Price? $p = d(20) = -0.025 \cdot 20^2 - 0.5 \cdot 20 + 60$

$= \boxed{40}$

Price: $p = d(25) = -0.025 \cdot 25^2 - 0.5 \cdot 25 + 60$

$= \boxed{40}$

**APPLIED EXAMPLE 6** Enclosing an Area The owner of Rancho Los Feliz has 3000 yards of fencing with which to enclose a rectangular piece of grazing land along the straight portion of a river. Fencing is not required along the river. Letting $x$ denote the width of the rectangle, find a function $f$ in the variable $x$ giving the area of the grazing land if she uses all of the fencing (Figure 25).




$A = x \cdot h$

$3000 = x + h + x = 2x + h$

(solve for h

$f(x) = x \cdot (h)$

$3000 = 2x + h$

$3000 - 2x = h \rightarrow f(x) = \boxed{x \cdot (3000 - 2x)}$

$= \boxed{-2x^2 + 3000x}$

Domain: $(0, 1500)$

**APPLIED EXAMPLE 7** Charter-Flight Revenue If exactly 200 people sign up for a charter flight, Leisure World Travel Agency charges \$300 per person. However, if more than 200 people sign up for the flight (assume that this is the case), then each fare is reduced by \$1 for each additional person. Letting $x$ denote the number of passengers above 200, find a function giving the revenue realized by the company.

assume $x \geq 0$, find $R(x)$

ppl fare

assume $x \geq 0$, find $R(x)$

$R(0) = \underset{ppl}{200} \cdot \underset{fare}{300} = 60000$

$R(1) = (200+1)(300-1) = 201 \cdot 299 = 60099$

$R(2) = (200+2)(300-2)$

assume we can transport 400 ppl

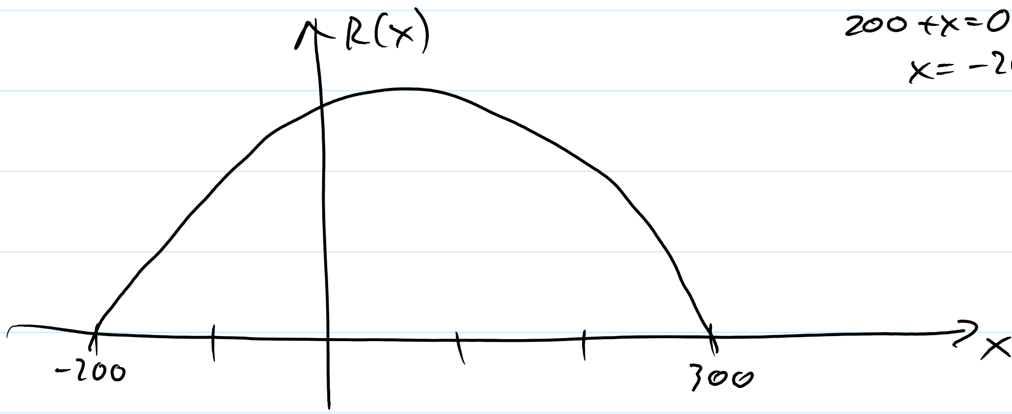
$R(x) = \boxed{(200+x)(300-x)}$ Domain $= [0, 200]$

For what $x$ is $R(x)$ maximized?

$R(x) = (200+x)(300-x) = 0$

$200 + x = 0$ $\quad$ $300 - x = 0$

$x = -200$ $\quad$ $x = 300$




Because $R(x) = -x^2 + 100x + 60000$ is a quadratic function, the maximum is in middle of $-200$ and $300$ so,

max is $\dfrac{300 + (-200)}{2} = \dfrac{100}{2} = \boxed{50}$

This corresponds to 250 passangers.

## 2.4 limits

$f(x) = 2x^2 - x$ $\qquad$ $8 - 2 = 6$

$f(x) = 2x - x$ $\qquad\qquad 8-2=6$

find $\dfrac{f(2+h)-f(2)}{h} = \dfrac{2\cdot(2+h)^2-(2+h)-\overbrace{[2\cdot 2^2-2]}^{}}{h}$

$= \dfrac{2(4+4h+h^2)-2-h-6}{h}$

$= \dfrac{\cancel{8}+8h+2h^2-\cancel{8}-h}{h} = \dfrac{7h+2h^2}{h}$

$g(h) = \dfrac{7h+2h^2}{h}$ what is domain of $g$?

all real numbers but $0$.

But $\dfrac{7h+2h^3}{h} = \dfrac{\cancel{h}(7+2h)}{\cancel{h}} = 7+2h$

has domain $(-\infty,\infty)$

So...

$g(h) \neq 7+2h$ ?

$\dfrac{\sqrt{4+h}-2}{h}$ if $h=0 \Rightarrow \dfrac{\sqrt{4}-2}{0} = \dfrac{2-2}{0} = \dfrac{0}{0}$

$\dfrac{\sqrt{4+h}-2}{h}\cdot\dfrac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \dfrac{\cancel{4}+h-\cancel{4}}{h(\sqrt{4+h}+2)} = \dfrac{1}{\sqrt{4+h}+2}$

$$\frac{\quad}{h} \cdot \frac{\quad}{\sqrt{4+h}+2} = \frac{\quad}{h\left(\sqrt{4+h}+2\right)} = \frac{\quad}{\sqrt{4+h}+2}$$

$$\text{if } h=0 \Rightarrow \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

is $\frac{0}{0} = \frac{1}{4}$ ?

$s(t) = 4t^2$ is a position of a particle for $t \geq 0$.

How do you find the instantaneous velocity at $t=1$?

average vel between $[a,b]$ is

$$\frac{s(b)-s(a)}{b-a}$$

Extra credit poblem is on Webassign is due before Monday's class.

Only two submissions.