

MTH 510

Homework 8
Due: March 21, 2019

Chapter 3: 16, 19, 22, 23, 26

Additional homework (suggestion: do these before the chapter homework).

1. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear map defined by

$$T(x, y) = (2x - y, -8x + 4y).$$

- (a) Find an isomorphism from the null space of T to \mathbb{R}^1
(b) Find an isomorphism from the range of T to \mathbb{R}^1

2. Let $T : \mathbb{R}^5 \rightarrow \mathbb{R}^4$ be the linear map defined by

$$T(x_1, x_2, x_3, x_4, x_5) = (x_1 + 3x_2 - 2x_3 - 3x_4, x_3 + 2x_4 + 3x_5, x_5, 2x_5).$$

- (a) Let $d = \dim \text{null } T$. Find an isomorphism from $\text{null } T$ to \mathbb{R}^d .
(b) Let $d = \dim \text{range } T$. Find an isomorphism from \mathbb{R}^d to $\text{range } T$.

3. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be defined by

$$T(x, y, z) = (3x + 2y + z, x + 5z).$$

Find

- (a) $\mathcal{M}(T)$
(b) $\mathcal{M}(T, ((1, 0, 0), (1, 1, 0), (1, 1, 1)), ((1, 0), (1, 1)))$

4. Let $T : \mathcal{P}_2(\mathbb{R}) \rightarrow \mathcal{P}_1(\mathbb{R})$ be defined by

$$T(p) = p'.$$

Find

- (a) $\mathcal{M}(T)$
(b) $\mathcal{M}(T, (1, x - 1, (x - 1)(x - 2)), (1, x - 1))$