MTH 510

Homework 7
Due: March 7, 2019

Chapter 3: Read Theorem 3.4. Do 9, 10, 11, 12.

Additional homework (suggestion: do these before the book homework):

1. Let U, V, W be vector spaces over $\mathbb{F}$ and let $A \in \mathcal{L}(V, W)$. Determine whether the function $T : \mathcal{L}(U, V) \to \mathcal{L}(U, W)$ defined by $T(X) = AX$ is a linear map.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map defined by

$$T(x, y) = (2x - y, -8x + 4y).$$

   (a) Find the null space of T.
   (b) Find the range of T.
   (c) Find a basis for the null space of T.
   (d) Find a basis for the range of T.

3. Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be the linear map defined by

$$T(x_1, x_2, x_3, x_4, x_5) = (x_1 + 3x_2 - 2x_3 - 3x_4, x_3 + 2x_4 + 3x_5, x_5, 2x_5).$$

   (a) Find the null space of T.
   (b) Find the range of T.
   (c) Find a basis for the null space of T.
   (d) Find a basis for the range of T.

4. Let A and B be nonempty sets and let $f : A \to B$ be a function. Prove

   (a) f is injective if and only if there exits a function $g : B \to A$ such that $g \circ f = id_A$.
   (b) f is surjective if and only if there exits a function $g : B \to A$ such that $f \circ g = id_B$.