Chapter 3: 9, 10, 11, 14, 15, 16

Additional homework (suggestion: do these before the chapter homework):

1. Let $U, V, W$ be vector spaces over $\mathbb{F}$ and let $A \in \mathcal{L}(V, W)$. Determine whether the function $T : \mathcal{L}(U, V) \rightarrow \mathcal{L}(U, W)$ defined by $T(X) = AX$ is a linear map.

2. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear map defined by
   
   $T(x, y) = (2x - y, -8x + 4y)$.

   (a) Find the null space of $T$.
   (b) Find the range of $T$.
   (c) Find a basis for the null space of $T$.
   (d) Find a basis for the range of $T$.

3. Let $T : \mathbb{R}^5 \rightarrow \mathbb{R}^4$ be the linear map defined by
   
   $T(x_1, x_2, x_3, x_4, x_5) = (x_1 + 3x_2 - 2x_3 - 3x_4, x_3 + 2x_4 + 3x_5, x_5, 2x_5)$.

   (a) Find the null space of $T$.
   (b) Find the range of $T$.
   (c) Find a basis for the null space of $T$.
   (d) Find a basis for the range of $T$.  

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