MTH 510

Homework 14
Due: May 2, 2019 (won't be collected)

Chapter 6: 3

Additional problems:

1. Consider the inner product space $\mathcal{P}_3(\mathbb{R})$ with inner product
$$\langle f, g\rangle = \int_0^1 f(x)g(x)dx.$$
Let $p(x) = 2x^3 + 3x + 1$ and $q(x) = 4x^2 - x + 7$. Find
   (a) $\langle p(x), q(x)\rangle$
   (b) $\|q(x)\|$

2. Which of the following lists are orthonormal with respect to the Euclidean inner product on $\mathbb{R}^3$.
   (a) $((\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}), (\frac{1}{3}, \frac{2}{3}, \frac{2}{3}))$
   (b) $((\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}))$
   (c) $((2, -2, 1), (2, 1, -2), (1, 2, 2))$
   (d) $((\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}))$

3. Consider the inner product space $\mathcal{P}_3(\mathbb{R})$ with inner product
$$\langle f, g\rangle = \int_0^1 f(x)g(x)dx$$
Let $p(x) = 2x^3 + 3x + 1$ and $q(x) = 4x^2 - x + 7$. Find the orthogonal decomposition of $p(x)$ with respect to $q(x)$.

4. Let $\mathbb{R}^3$ have the Euclidean inner product. Use the Gram-Schmidt procedure to transform the basis $((1, 1, 1), (1, 1, 0), (1, 0, 0))$ for $\mathbb{R}^3$ into an orthonormal basis.

5. Let $\mathbb{R}^3$ have the inner product $< \mathbf{u}, \mathbf{v} >= u_1v_1 + 2u_2v_2 + 3u_3v_3$. Use the Gram-Schmidt procedure to transform the basis $((1, 1, 1), (1, 1, 0), (1, 0, 0))$ for $\mathbb{R}^3$ into an orthonormal basis.