Problem 1. Let $S \subset \mathbb{R}^3$ be a compact oriented 2-manifold with boundary parametrized by $r : U \to S$, where $U$ is a bounded domain in $\mathbb{R}^2$. Let $n = (a, b, c)$ be a field of unit normal vectors on $S$. Prove that
\[
\int_S a \, dy \wedge dz + b \, dz \wedge dx + c \, dx \wedge dy = \pm \text{area}(S).
\]

Problem 2. Let $X$ be a compact $n$-manifold with boundary, $\alpha \in \Omega^r(X)$ and $\beta \in \Omega^s(X)$ with $r + s = n - 1$. Prove the generalized integration by parts formula
\[
(-1)^r \int_X \alpha \wedge d\beta = \int_{\partial X} \alpha \wedge \beta - \int_X d\alpha \wedge \beta.
\]

Problem 3. Find the simple closed curve $C \subset \mathbb{R}^2$ for which the value of the line integral
\[
\oint_C (y^3 - y) \, dx - 2x^3 \, dy
\]
is a maximum.

Problem 4. Let $\omega = xz^2 \, dy \wedge dz + \left(\frac{1}{3} y^3 + \tan z\right) \, dz \wedge dx + (x^2 z + y^2) \, dx \wedge dy$ be a differential 2–form on $\mathbb{R}^3$.

(1) Evaluate the difference
\[
\int_{S_1} \omega - \int_{S_2} \omega,
\]
where $S_1$ is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and $S_2$ is the unit disk $x^2 + y^2 \leq 1$ in the $xy$–plane. Both $S_1$ and $S_2$ are oriented so that the boundary circle $x^2 + y^2 = 1$ in the $xy$–plane is oriented counterclockwise when looked at from the top of the $z$–axis.

(2) Is there a differential 1–form $\alpha$ on $\mathbb{R}^3$ such that $\omega = d\alpha$? Explain.