PRACTICE PROBLEMS FOR TEST I

1. Find the solutions of $y' = (y+1)^2$ satisfying the initial conditions (a) $y(0) = 1$; (b) $y(0) = -1$. What are the longest intervals of existence for each of these solutions ?

2. Find the general solutions of the following equations : (a) $y' + ty = t$; (b) $(t^2 + t - 2)y' - y = 0,\ t > 1$; (c) $ty' - y = t^3 \cos t,\ t > 0$, (d) $(t+1)y' + ty = e^{-t},\ t > -1$.

3. A particle moves along the x–axis in such a way that at any time t its velocity is $x^2 + t$. It is located at $x = 1$ at time $t = 1$. What is its velocity and acceleration at that time ?

4. Use isoclines to plot solution curves for $y' = y^2 - t$. Do the solution curves ever intersect ? Explain.

5. The following questions concern the equation $y' = y^2 - 2y$.

(a) If $y(0) = 1.9$ what happens to $y(t)$ for large t ?

(b) If $y(0) = 1.9$, does the graph of $y(t)$ have an inflection point ? If so, for what value of y ?

(c) Find y' and y'' at the moment when $y = 3$.

6. Use Euler's method with step $h = 0.1$ to find a numerical approximation to $x(0.5)$, the solution of $x' = -x + t$ with $x(0) = 0$. Compute approximations to 3 decimal places.

7. Suppose a 20–gallon tank contains 10 gallons of clean water at time $t = 0$. Water containing 0.3 pounds per gallon of salt enters the tank at a rate of 2 gallons per minute. Suppose clean water enters the tank at a rate of 1 gallon per minute. Finally, suppose the liquid in the tank is well mixed at all times and saltwater exits the tank at a rate of 0.5 gallons per minute. Find the concentration of salt at the moment when the tank starts to overflow.

8. Solve the differential equation $y' = 1 + t\,e^{-y}$ by using the substitution $u = e^y$.

ANSWERS

1. (a) $y = -1 + 1/(0.5 - t),\ -\infty < t < 0.5;$ (b) $y = -1,\ -\infty < t < \infty.$

2. (a) $y = 1 + k\, e^{-t^2/2},$ (b) $y = k \left(\dfrac{t-1}{t+2}\right)^{1/3},$ (c) $y = t^2 \sin t + t \cos t + Ct,$ (d) $y = (Ct + C - 1)e^{-t}.$

3. Velocity is 2, acceleration is 5.

5. (a) Approaches $y = 0;$ (b) for $y = 1;$ (c) $y' = 3$ and $y'' = 12.$

6. 0.090.

7. $(8 - 4^{4/5})/40 \approx 0.1242$

8. $y = \ln(\, Ce^t - t - 1).$