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Corrigendum

Corrigendum to "Modeling the transmission dynamics and control of Hepatitis B virus in China" [J. Theor. Biol. 262 (2010) 330–338]

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In this note, we provide corrections to a theorem and its proof in our paper Zou et al. (2010) and make some additional remarks.

1. Corrections

Theorem 2 in our paper Zou et al. (2010) states that *the endemic equilibrium E^* of model* (2.1) *is a stable node if it exists.* Both the statement and the proof were incorrect. We realized the mistake right after the publication of the paper and a correction of the statement and its proof was presented in the Ph.D. Thesis of the first author in March 2010 (Zou, 2010, Theorem 5.1.2, pp. 65–66 and Appendix III, p. 95). The following corrections are adapted from Zou (2010).

In order to discuss the steady states of model (2.1) of Zou et al. (2010), we only need to consider the following system, which is (3.1) in Zou et al. (2010):

$$\begin{cases} \dfrac{dS}{dt} = \mu\omega(1-\nu C)+\psi V-(\mu_0+\beta I+\varepsilon\beta C+\gamma_3)S, \\ \dfrac{dL}{dt} = (\beta I+\varepsilon\beta C)S-(\mu_0+\sigma)L, \\ \dfrac{dI}{dt} = \sigma L-(\mu_0+\gamma_1)I, \\ \dfrac{dC}{dt} = \mu\omega\nu C+q\gamma_1 I-(\mu_0+\mu_1+\gamma_2)C, \\ \dfrac{dV}{dt} = \mu(1-\omega)+\gamma_3 S-(\mu_0+\psi)V. \end{cases}$$

As defined in (2.2) of Zou et al. (2010), the basic reproduction number is given by

$$R_0 = \frac{\mu(\psi+\mu_0\omega)}{\mu_0(\mu_0+\gamma_3+\psi)}\frac{\sigma\beta}{(\mu_0+\gamma_1)(\mu_0+\sigma)}\left[1+\frac{q\gamma_1\varepsilon}{\mu_0+\mu_1+\gamma_2-\mu\omega\nu}\right].$$

It is proved in Zou et al. (2010) that the system has an endemic equilibrium $E^*=(S^*,L^*,I^*,C^*,V^*)$ if $R_0>1$, where

$$S^* = \frac{(\mu_0+\mu_1+\gamma_2-\mu\omega\nu)(\mu_0+\sigma)(\gamma_1+\mu_0)}{(\mu_0+\mu_1+\gamma_2+\varepsilon q\gamma_1-\mu\omega\nu)\beta\sigma} = \frac{S_0}{R_0},$$

$$L^* = \frac{\mu_0(\gamma_3+\mu_0+\psi)S^*(\mu_0+\mu_1+\gamma_2)(\gamma_1+\mu_0)(R_0-1)}{(\mu_0+\psi)\sigma[\beta S^*(\mu_0+\mu_1+\gamma_2+\varepsilon q\gamma_1-\mu\omega\nu)+\mu\omega\nu q\gamma_1]},$$

$$I^* = \frac{\mu_0 S^*(\gamma_3+\mu_0+\psi)(R_0-1)(\mu_0+\mu_1+\gamma_2-\mu\omega\nu)}{(\mu_0+\psi)[\beta S^*(\mu_0+\mu_1+\gamma_2+\varepsilon q\gamma_1-\mu\omega\nu)+\mu\omega\nu q\gamma_1]},$$

$$C^* = \frac{\mu_0\varepsilon\beta q\gamma_1(\mu_0+\gamma_1)(\gamma_3+\mu_0+\psi)(S^*)^2(R_0-1)}{(\mu_0+\psi)[\beta S^*(\mu_0+\mu_1+\gamma_2+\varepsilon q\gamma_1-\mu\omega\nu)+\mu\omega\nu q\gamma_1]},$$

$$V^* = \frac{\mu(1-\omega)+\gamma_3 S^*}{\mu_0+\psi}.$$

Theorem 2 in Zou et al. (2010) should be replaced by the following result.

Theorem 2. *Assume that $R_0>1$. Then the endemic equilibrium E^* is locally asymptotically stable if*

(i) $b_1b_2-b_3>0$,
(ii) $b_3(b_1b_2-b_3)+b_1(b_5-b_1b_4)>0$,
(iii) $(b_1b_2-b_3)(b_3b_4-b_2b_5)-(b_5-b_1b_4)^2>0$,

where b_is are given as follows

$$\begin{aligned} b_1 &= \frac{\sigma L^*}{I^*}+\frac{q\gamma_1 I^*}{L^*}+\frac{(\beta I^*+\epsilon\beta C^*)S^*}{L^*}+\psi+\beta I^*+\varepsilon\beta C^*+\gamma_3+2\mu_0, \\ b_2 &= \frac{1}{L^*I^*C^*}[I^*S^*C^*(I^*+C^*\varepsilon)^2\beta^2+(S^*\varepsilon C^*I^{*2}q\gamma_1+2\mu_0S^*I^{*2}C^* \\ &\quad +\varepsilon C^{*2}L^{*2}\sigma+S^*I^{*3}q\gamma_1+S^*\varepsilon C^{*2}\sigma L^*+\varepsilon C^{*2}\mu_0L^*I^*+I^{*3}L^*q\gamma_1 \\ &\quad +\varepsilon C^{*2}\psi L^*I^*+\gamma_3S^*I^{*2}C^*+\gamma_3S^*\varepsilon C^{*2}I^*+\varepsilon C^*L^*I^{*2}q\gamma_1 \\ &\quad +I^{*2}\psi L^*C^*+\psi S^*I^{*2}C^*+\psi S^*\varepsilon C^{*2}I^*+2\mu_0S^*\varepsilon C^{*2}I^*+I^{*2}\mu_0L^*C^* \\ &\quad +I^*L^{*2}\sigma C^*)\beta+\gamma_3L^{*2}\sigma C^*+\mu_0^2L^*I^*C^*+2\mu_0L^{*2}\sigma C^*+\psi L^{*2}\sigma C^* \\ &\quad +\gamma_3\mu_0L^*I^*C^*+\mu_0\psi L^*I^*C^*+2\mu_0L^*I^{*2}q\gamma_1 \\ &\quad +\gamma_3L^*I^{*2}q\gamma_1+L^{*2}\sigma q\gamma_1I^*+\psi L^*I^{*2}q\gamma_1], \\ b_3 &= \frac{1}{L^*I^*C^*}(\mu_0^2L^{*2}\sigma C^*+2\mu_0L^{*2}\sigma q\gamma_1I^*+\mu_0^2\beta S^*I^{*2}C^* \\ &\quad +2\beta^2I^{*2}\mu_0S^*\varepsilon C^{*2}+\beta I^*\psi L^{*2}\sigma C^*+\beta I^{*3}\psi L^*q\gamma_1 \\ &\quad +\varepsilon\beta C^*\psi L^*I^{*2}q\gamma_1+\varepsilon^2\beta^2C^{*3}S^*\sigma L^*+\varepsilon^2\beta^2C^{*2}S^*I^{*2}q\gamma_1 \\ &\quad +\varepsilon\beta C^*L^{*2}\sigma q\gamma_1I^*+\beta I^{*2}L^{*2}\sigma q\gamma_1+\mu_0^2\beta S^*\varepsilon C^{*2}I^*+\varepsilon^2\beta^2C^{*3}\mu_0S^*I^* \\ &\quad +\varepsilon\beta C^{*2}\mu_0L^{*2}\sigma+\mu_0\psi\beta S^*I^{*2}C^*+\mu_0\psi\beta S^*\varepsilon C^{*2}I^*+\mu_0\psi L^{*2}\sigma C^* \\ &\quad +\mu_0\psi L^*I^{*2}q\gamma_1+\varepsilon\beta C^*\mu_0L^*I^{*2}q\gamma_1+\varepsilon^2\beta^2C^{*3}\psi S^*I^* \end{aligned}$$

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$$
\begin{aligned}
&+\varepsilon\beta C^{*2}\psi L^{*2}\sigma+\gamma_3\mu_0\beta S^* I^{*2}C^*+\gamma_3\mu_0\beta S^*\varepsilon C^{*2}I^*+\gamma_3\mu_0 L^{*2}\sigma C^*\\
&+\gamma_3\mu_0 L^* I^{*2}q\gamma_1+\mu_0^2 L^* I^{*2}q\gamma_1+\beta^2\sigma L I^{*2}S^*C^*+\beta^2 I^{*3}\mu_0 S^*C^*\\
&+\gamma_3\beta S^* I^{*3}q\gamma_1+\gamma_3\beta S^*\varepsilon C^{*2}\sigma L^*+\gamma 3\beta S^*\varepsilon C^* I^{*2}q\gamma_1+\psi\beta S^* I^{*3}q\gamma_1\\
&+\gamma_3 L^{*2}\sigma q\gamma_1 I^*+\psi\beta S^*\varepsilon C^* I^{*2}q\gamma_1+\psi\beta S^*\varepsilon C^{*2}\sigma L^*\\
&+\psi L^{*2}\sigma q\gamma_1 I^*+\beta I^*\mu_0 L^{*2}\sigma C^*+\beta I^{*3}\mu_0 L^* q\gamma_1+\beta^2 I^{*3}\psi S^*C^*\\
&+\beta^2 I^{*4}S^*q\gamma_1+2\beta^2 I^{*2}\psi S^*\varepsilon C^{*2}+2\mu_0\beta S^* I^{*3}q\gamma_1+2\mu_0\beta S^*\varepsilon C^{*2}\sigma L^*\\
&+2\mu_0\beta S^*\varepsilon C^* I^{*2}q\gamma_1+2\beta^2 I^* S^*\varepsilon C^{*2}\sigma L^*+2\beta^2 I^{*3}S^*\varepsilon C^* q\gamma_1),
\end{aligned}
$$

$$
\begin{aligned}
b_4=&\frac{1}{L^*I^*C^*}(2\beta^2\sigma L^{*2}q\gamma_1 C^*\varepsilon S^*+2\beta^2 I^*\psi S^*\varepsilon C^{*2}\sigma L^*+2\beta^2 I^{*3}\psi S^*\varepsilon C^* q\gamma_1\\
&+2\beta^2 I^*\mu_0 S^*\varepsilon C^{*2}\sigma L^*+\beta^2\sigma L^* I^{*3}S^*q\gamma_1+\beta^2 I^{*4}\psi S^*q\gamma_1\\
&+\beta\sigma L^* I^{*2}q\gamma_1 C^*\mu\omega v+\beta^2\sigma L^* I^*\varepsilon^2 C^{*2}q\gamma_1 S^*+\beta\sigma L^* I^*\varepsilon C^{*2}q\gamma_1\mu\omega v\\
&+\beta I^{*2}\mu_0 L^{*2}\sigma q\gamma_1+\beta I^{*2}\psi L^{*2}\sigma q\gamma_1+\mu_0^2\beta S^* I^{*3}q\gamma_1+\beta^2 I^{*4}\mu_0 S^* q\gamma_1\\
&+\mu_0^2\beta S^*\varepsilon C^{*2}\sigma L^*+\mu_0^2\beta S^*\varepsilon C^* I^{*2}q\gamma_1+\varepsilon\beta C^*\psi L^{*2}\sigma q\gamma_1 I^*\\
&+\gamma_3\mu_0\beta S^* I^{*3}q\gamma_1+\mu_0^2 L^{*2}\sigma q\gamma_1 I^*+\varepsilon^2\beta^2 C^{*3}\mu_0 S^*\sigma L^*\\
&+\varepsilon^2\beta^2 C^{*2}\mu_0 S^* I^{*2}q\gamma_1+\varepsilon\beta C^*\mu_0 L^{*2}\sigma q\gamma_1 I^*+\varepsilon^2\beta^2 C^{*3}\psi S^*\sigma L^*\\
&+\varepsilon^2\beta^2 C^{*2}\psi S^* I^{*2}q\gamma_1+\mu_0\psi L^{*2}\sigma q\gamma_1 I^*\\
&+\gamma_3\mu_0\beta S^*\varepsilon C^{*2}\sigma L^*+\gamma_3\mu_0\beta S^*\varepsilon C^* I^{*2}q\gamma_1\\
&+\gamma_3\mu_0 L^{*2}\sigma q\gamma_1 I^*+\mu_0\psi\beta S^* I^{*3}q\gamma_1+\mu_0\psi\beta S^*\varepsilon C^{*2}\sigma L^*\\
&+\mu_0\psi\beta S^*\varepsilon C^* I^{*2}q\gamma_1+\beta^2\sigma L^* I^{*2}\mu_0 S^*C^*\\
&+\beta^2\sigma L^* I^{*2}\psi S^*C^*+2\beta^2 I^{*3}\mu_0 S^*\varepsilon C^* q\gamma_1),
\end{aligned}
$$

$$
\begin{aligned}
b_5=&\frac{1}{L^*I^*C^*}(\beta\sigma L^* I^*\varepsilon C^{*2}\psi q\gamma_1\mu\omega v+\beta^2\sigma L^* I^*\varepsilon^2 C^{*2}\psi q\gamma_1 S^*\\
&+\beta\sigma L^* I^*\varepsilon C^{*2}\mu_0 q\gamma_1\mu\omega v+\beta^2\sigma L^* I^*\varepsilon^2 C^{*2}\mu_0 q\gamma_1 S^*\\
&+\beta^2\sigma L^* I^{*3}\psi S^* q\gamma_1+\beta^2\sigma L^* I^{*3}\mu_0 S^* q\gamma_1+\beta\sigma L^* I^{*2}\mu_0 q\gamma_1 C^*\mu\omega v\\
&+2\beta^2\sigma L^* I^{*2}\mu_0 q\gamma_1 C^*\varepsilon S^*+2\beta^2\sigma L^* I^{*2}\psi q\gamma_1 C^*\varepsilon S^*\\
&+\beta\sigma L^* I^{*2}\psi q\gamma_1 C^*\mu\omega v).
\end{aligned}
$$

Proof. The Jacobian matrix of the system at E^* is

$$
J(E^*)=\begin{pmatrix}
-\mu_0-\beta I^*-\varepsilon\beta C^*-\gamma_3 & 0 & -\beta S^* & -\mu\omega v-\varepsilon\beta S^* & \psi\\
\beta I^*+\varepsilon\beta C^* & -\mu_0-\sigma & \beta S^* & \varepsilon\beta S^* & 0\\
0 & \sigma & -\mu_0-\gamma_1 & 0 & 0\\
0 & 0 & q\gamma_1 & \mu\omega v-\mu_0-\mu_1-\gamma_2 & 0\\
\gamma_3 & 0 & 0 & 0 & -\mu_0-\psi
\end{pmatrix}.
$$

Its eigenvalues satisfy the following characteristic equation:

$$\Phi^*(\lambda):=\lambda^5+b_1\lambda^4+b_2\lambda^3+b_3\lambda^2+b_4\lambda+b_5=0.$$

By Routh–Hurwitz criterion, we know that every root of $\Phi^*(\lambda)=0$ has negative real part if and only if every determinant of the matrix H_i is positive, $i=1,\ldots,5$, where

$$
H_1=(b_1),\quad H_2=\begin{pmatrix} b_1 & 1\\ b_3 & b_2\end{pmatrix},\quad H_3=\begin{pmatrix} b_1 & 1 & 0\\ b_3 & b_2 & b_1\\ b_5 & b_4 & b_3\end{pmatrix},
$$

$$
H_4=\begin{pmatrix} b_1 & 1 & 0 & 0\\ b_3 & b_2 & b_1 & 1\\ b_5 & b_4 & b_3 & b_2\\ 0 & 0 & b_5 & b_4\end{pmatrix},\quad H_5=\begin{pmatrix} b_1 & 1 & 0 & 0 & 0\\ b_3 & b_2 & b_1 & 1 & 0\\ b_5 & b_4 & b_3 & b_2 & b_1\\ 0 & 0 & b_5 & b_4 & b_3\\ 0 & 0 & 0 & 0 & b_5\end{pmatrix}.
$$

Note that all $b_i>0$ $(i=1,\ldots,5)$ since all parameters are positive. Hence, condition (i) of the theorem implies that the determinant of H_2 is positive; condition (ii) ensures that the determinant of H_3 is positive; and condition (iii) guarantees that the determinant of H_4 is positive. Finally, the determinant of H_5 is positive if and only if

$$b_5[(b_1b_2-b_3)(b_3b_4-b_2b_5)-(b_5-b_1b_4)^2]>0,$$

which follows from the fact that $b_5>0$ and condition (iii). The conclusion thus follows. □

2. Remarks

The conditions (i)–(iii) can also be expressed as

$$
\begin{cases}
D:=(b_5-b_1b_4)^2(b_3^2-4b_1b_5)>0,\\
\max\left\{\dfrac{b_3}{b_1},\dfrac{b_3^2-b_1(b_5-b_1b_4)}{b_1b_3},\dfrac{b_3(b_1b_4+b_5)-\sqrt{D}}{2b_1b_5}\right\}\\
<b_2<\dfrac{b_3(b_1b_4+b_5)+\sqrt{D}}{2b_1b_5}.
\end{cases}
$$

Define $N(t):=S(t)+L(t)+I(t)+C(t)+R(t)+V(t)$. We obtain that

$$\frac{dN(t)}{dt}=\mu-\mu_0N(t)-\mu_1C(t),$$

which implies that

$$N(t)\le\frac{\mu}{\mu_0}+e^{-\mu_0t}\left(N(t_0)-\frac{\mu}{\mu_0}\right)\to\frac{\mu}{\mu_0}$$

as $t\to+\infty$. Thus, for any choice of parameter values satisfying our assumptions, $(S(t), L(t), I(t), C(t), R(t), V(t))$ will fall into the set $\{(S, L, I, C, R, V)\in R_+^6 : 0<S+L+I+C+R+V\le\mu/\mu_0\}$ in a finite time t.

3. HBV is endemic in China

We would like to point out that in Section 5 in Zou et al. (2010), we numerically simulated the data on acute hepatitis B reported by the Ministry of Chinese Health from 2003 to 2008 using the parameter values in Table 1. Since our simulation matched the reported data well, based on the model and the used parameter values, the basic reproduction number was estimated to be $R_0=2.406$. We then stated: "This indicates that hepatitis B is endemic in mainland China: it stabilizes and is

Table 1
Reported hepatitis B data in China, 2009–2010 (MOHC, 2012).

Year	2009	2010	2011
Cases	1 179 607	1 060 582	1 093 335

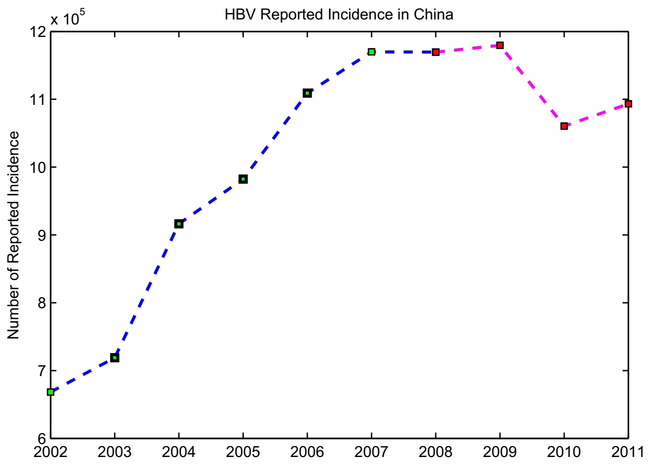

Fig. 1. Hepatitis B data reported by the Ministry of Health of China from 2002 to 2011.

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approaching its equilibrium.'' This conclusion (and those proposed control strategies) was made based on the evaluation and sensitivity analysis of the basic reproduction number and numerical simulations using reported data while the stability of the positive equilibrium E^* was never used.

The hepatitis B data reported by the Ministry of Chinese Health from 2009 to 2011 are available now (MOHC, 2012), see Table 1.

Fig. 1(a) in Zou et al. (2010) showed the hepatitis B data reported by the Ministry of Health of China from 2003 to 2008. Adding the data of 2002 and new data from 2009 to 2011 in Table 1 into Fig. 1(a) in Zou et al. (2010), we obtain the incidence numbers of HBV in China since 2002 (Fig. 1), which further confirms our conclusion that hepatitis B is endemic in mainland China: it stabilizes and is approaching its equilibrium.

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