Math 161 Lab 9/16

1. Calculate the derivatives of the following functions:

   a. $f(x) = 3x^2 - \sin x$

   $$f'(x) = 6x - \cos x$$

   b. $f(x) = \dfrac{1}{\sqrt{x}} = x^{-1/2}$

   $$f'(x) = -\tfrac{1}{2}x^{-3/2} = \frac{-1}{2\sqrt{x^3}}$$

   c. $f(x) = \dfrac{x^3}{\cos x}$

   $$f'(x) = \frac{\cos(x)\cdot 3x^2 + x^3\sin(x)}{\cos^2 x} = \frac{3x^2}{\cos x} + \frac{x^3\tan(x)}{\cos x} = 3x^2\sec(x) + x^3\sec(x)\tan(x)$$

2. If g(x) is differentiable find an expression for the derivatives of:

   a. $f(x) = x \cdot g(x)$

   $$f'(x) = g(x) + xg'(x)$$

   b. $f(x) = \dfrac{x}{g(x)}$

   It is ok to stop here.

   $$f'(x) = \frac{g(x) - xg'(x)}{(g(x))^2} = \frac{1}{g(x)} - \frac{xg'(x)}{(g(x))^2}$$

   c. $f(x) = \dfrac{g(x)}{x}$

   $$f'(x) = \frac{xg'(x) - g(x)}{x^2}$$

3. Use the Quotient Rule to prove that:

a. $\frac{d}{dx}(\cot x) = -\csc^2 x$

$$\frac{d}{dx}(\cot(x)) = \frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-1}{\sin^2(x)} = -\csc^2(x)$$

b. $\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$

$$\frac{d}{dx}(\csc(x)) = \frac{d}{dx}\left((\sin(x))^{-1}\right) = -1(\sin(x))^{-2} \cdot \cos(x) = \frac{-\cos(x)}{\sin^2(x)} = -\csc(x)\cot(x)$$

4. Use the Product Rule twice to compute the derivative of $f(x) = x^3 \cdot \sin x \cdot \cos x$

$$f'(x) = 3x^2\underbrace{(\sin x \cos x)}_{\frac{1}{2}\sin(2x)} + x^3\underbrace{(\cos^2 x - \sin^2 x)}_{\cos(2x)}$$

$$= \frac{3}{2}x^2 \sin(2x) + x^3\cos(2x)$$

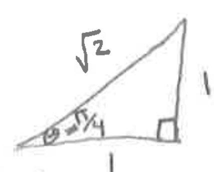
5. Find the first and second derivatives of $f(t) = t^3 \cdot \cos t$

$$f'(t) = 3t^2\cos t - t^3 \sin t$$

$$f''(t) = 6t\cos t - 3t^2\sin t - (3t^2\sin t + t^3\cos t)$$

$$= (6t - t^3)\cos t - 6t^2 \sin t$$

6. Find the equation of the tangent line and normal line to $f(x) = \tan x$ at $\left(\frac{\pi}{4}, 1\right)$



$$f'(x) = \sec^2(x)$$

$$f'(\pi/4) = \sec^2(\pi/4) = \frac{1}{\cos^2(\pi/4)} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = 2$$

tangent line

$$\boxed{y - 1 = 2\left(x - \frac{\pi}{4}\right)}$$

normal line

$$\boxed{y - 1 = -\frac{1}{2}(x - \pi/4)}$$