The exam will consist of **about nine** questions (which may be further divided into **several parts**), with **unequal** weights. You are expected to know how to do the following types of problems (this list is by no means exhaustive, I just want to give you an idea of what you should expect in the exam. For further practice, do the recommended questions or the problems in the book):

1. **Inverse functions, log, exponential, trigonometric functions, and their calculus**

- Find $(f^{-1})'(a)$. $f(x) = x^3 + 3\sin x + 2\cos x, \quad a = 2$
- Use the Laws of Logarithms to expand the quantity.

  $\ln \sqrt[3]{\frac{x-1}{x+1}}$
- Express the quantity as a single logarithm.

  $\ln 3 + \frac{1}{3}\ln 8$
- Use the Laws of Logarithms to expand the quantity.

  $\ln s^4 \sqrt{t\sqrt{u}}$
- Differentiate the function.

  $y = \ln|2 - x - 5x^2|$

  $g(x) = \ln\left(x\sqrt{x^2 - 1}\right)$
- Use logarithmic differentiation to find the derivative of the function.

  $y = \sqrt{\frac{x-1}{x^4+1}}$

- Evaluate the integral.

$$\int_1^2 \frac{dt}{8-3t}$$

$$\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$$

$$\int_e^6 \frac{dx}{x \ln x}$$

$$\int_1^e \frac{x^2 + x + 1}{x}\, dx$$

- Differentiate the function.

$$y = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

$$y = x^{\cos x}$$

$$y = \sqrt{x}^{\,x}$$

- Evaluate the integral.

$$\int_0^1 \frac{\sqrt{1 + e^{-x}}}{e^x}\, dx \qquad \int \frac{2^x}{2^x + 1}\, dx$$

- Find the exact value of each expression.

$\sin^{-1}(\sqrt{3}/2)$

$\tan^{-1}(1/\sqrt{3})$

arctan 1

- Find the derivative of the function.

  $y = \tan^{-1}(x^2)$

  $y = \sin^{-1}(2x + 1)$

- Evaluate the integral.

  $$\int_0^{\sqrt{3}/4} \frac{dx}{1 + 16x^2}$$

  $$\int \frac{t^2}{\sqrt{1 - t^6}}\, dt$$

  $$\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1 - x^2}}\, dx$$

- Find the derivative.

  $\cosh(\ln x)$

  $\sinh^{-1}(\tan x)$

- Evaluate the integral.

  $$\int \frac{\cosh x}{\cosh^2 x - 1}\, dx$$

  $$\int_4^6 \frac{1}{\sqrt{t^2 - 9}}\, dt$$

  $$\int \frac{e^x}{1 - e^{2x}}\, dx$$

2. L’hospital Rule

- Find the limit.

$$\lim_{x \to (\pi/2)^+} \frac{\cos x}{1 - \sin x}$$

$$\lim_{t \to 0} \frac{e^{2t} - 1}{\sin t}$$

$$\lim_{x \to \infty} \sqrt{x}\, e^{-x/2}$$

3. Integration by parts, Trigonometric integration/substitution

- Evaluate the integral.

$$\int \sin^{-1} x \, dx$$

$$\int \cos \sqrt{x} \, dx$$

$$\int_1^4 e^{\sqrt{x}} \, dx$$

$$\int \sin^2 x \cos^3 x \, dx$$

- Evaluate the integral using trigonometric substitution.

$$\int \frac{dx}{x^2 \sqrt{4 - x^2}}$$

$$\int \frac{x}{\sqrt{1 + x^2}} \, dx$$ (You may need: (sec x)'= sec x tan x)

$$\int_0^1 \sqrt{x^2+1}\,dx$$

$$\int_0^1 \frac{dx}{(x^2+1)^2}$$

[You may need the formula: $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$]

4. Partial Fractions

- Write out the form of the partial fraction decomposition of the function . Do not determine the numerical values of the coefficients.

  (a) $\dfrac{t^6+1}{t^6+t^3}$ (b) $\dfrac{x^5+1}{(x^2-x)(x^4+2x^2+1)}$

- Evaluate the integral.

  $$\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$$

5. Improper integrals

- Use the Comparison Theorem to determine whether the integral is convergent or divergent.

  $$\int_0^\infty \frac{x}{x^3+1}\,dx$$

  $$\int_1^\infty \frac{2+e^{-x}}{x}\,dx$$

- Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_1^{\infty} \frac{\ln x}{x}\, dx$$

$$\int_0^{\infty} \frac{e^x}{e^{2x} + 3}\, dx$$

$$\int_0^{5} \frac{w}{w - 2}\, dw$$

6. Power Series

- Find a power series representation for the function

  $$f(x) = \frac{1}{x + 10}$$

- Find the Taylor Series of f centered at x=0:

  $f(x) = \sinh x$

- Find the Taylor Series of f centered at x=a:

  $f(x) = \cos x, \quad a = \pi$

- Evaluate the indefinite integral as an infinite series.

  $$\int \frac{e^x - 1}{x}\, dx$$

- Use the first four non-zero terms of the Taylor series to find an approximate value of the integral:

  $$\int_0^1 x \cos(x^3)\, dx$$

- Use series to evaluate the limit.

$$\lim_{x \to 0} \frac{x - \ln(1 + x)}{x^2}$$

$$\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5} \qquad \lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x}$$

7. Polar coordinates, parametric curves

- Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

  $x = \sin^3\theta, \quad y = \cos^3\theta; \quad \theta = \pi/6$

- Find the area enclosed by the x-axis and the curve $x = 1 + e^t, y = t - t^2$.
- Find the exact length of the curve.

  $x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leqslant t \leqslant 1$

- Find dy/dx.

  $x = t \sin t, \quad y = t^2 + t$

- Identify the curve by finding a Cartesian equation for the curve.

  $r = 2 \cos \theta$

- Find a polar equation for the curve represented by the given Cartesian equation.

  $y = 1 + 3x$

- Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

  $r = 2 - \sin \theta, \quad \theta = \pi/3$

- Find the area of the region enclosed by one loop of the curve.

  $r = 4 \cos 3\theta$

  $r^2 = \sin 2\theta$

- Find the exact length of the polar curve.

  $r = 3 \sin \theta, \quad 0 \leqslant \theta \leqslant \pi/3$

  $r = e^{2\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$