Ex. 5.2
25)

$$
\begin{aligned}
g^{\prime}(x) & =\left[\ln \left(x \sqrt{x^{2}-1}\right)\right]^{\prime} \\
& =\left[\ln x+\ln \sqrt{x^{2}-1}\right]^{\prime} \\
& =\left[\ln x+\frac{1}{2} \ln \left(x^{2}-1\right)\right]^{\prime} \\
& =\frac{1}{x}+\frac{1}{2} \cdot \frac{\left(x^{2}-1\right)^{\prime}}{x^{2}-1} \\
& =\frac{1}{x}+\frac{1}{2} \cdot \frac{2 x}{x^{2}-1} \\
& =\frac{1}{x}+\frac{x}{x^{2}-1}
\end{aligned}
$$

29) 

$$
\begin{aligned}
U \operatorname{sing}(\ln |f|)^{\prime} & =\frac{f^{\prime}}{f}, \\
y^{\prime}=\left[\ln \left|2-x-5 x^{2}\right|\right]^{\prime} & =\frac{\left(2-x-5 x^{2}\right)}{2-x-5 x^{2}} \\
& =\frac{-1-10 x}{2-x-5 x^{2}}
\end{aligned}
$$

53) $y=\sqrt{\frac{x-1}{x^{4}+1}}=\left(\frac{x-1}{x^{4}+1}\right)^{\frac{1}{2}}$

$$
\therefore \ln y=\frac{1}{2} \ln \left(\frac{x-1}{x^{4}+1}\right)=\frac{1}{2}\left[\ln (x-1)-\ln \left(x^{4}+1\right)\right]
$$

$$
\begin{aligned}
\frac{y^{\prime}}{y} & =\frac{1}{2}\left[\frac{(x-1)^{\prime}}{x-1}-\frac{\left(x^{4}+1\right)^{\prime}}{x^{4}+1}\right] \\
& =\frac{1}{2}\left[\frac{1}{x-1}-\frac{4 x^{3}}{x^{4}+1}\right] \\
\therefore y^{\prime} & =\frac{y}{2}\left[\frac{1}{x-1}-\frac{4 x^{3}}{x^{4}+1}\right] \\
& =\sqrt{\frac{x-1}{x^{4}+1}}\left[\frac{1}{x-1}-\frac{4 x^{3}}{x^{4}+1}\right] \\
& =\frac{1}{\sqrt{\left(x^{4}+1\right)(x-1)}}-\frac{4 x^{3} \sqrt{x-1}}{\left(x^{4}+1\right)^{\frac{3}{2}}}
\end{aligned}
$$

57) 

$$
\begin{aligned}
& \int_{1}^{e} \frac{x^{2}+x+1}{x} d x \\
= & \int_{1}^{e}\left(x+1+\frac{1}{x}\right) d x \\
= & {\left[\frac{x^{2}}{2}+x+\ln |x|\right]_{1}^{e} } \\
= & \left(\frac{e^{2}}{2}+e+\ln e\right)-\left(\frac{1^{2}}{2}+1+\ln 1\right)(e>0) \\
= & \left(\frac{e^{2}}{2}+e+1\right)-\left(\frac{1}{2}+1+0\right) \\
= & \frac{e^{2}}{2}+e-\frac{1}{2} \#
\end{aligned}
$$

Recommended
Ex.5.2 (15)

$$
\begin{aligned}
f^{\prime}(x) & =\frac{1}{2} x^{-\frac{1}{2}} \ln x+\frac{\sqrt{x}}{x} \\
& =\frac{\ln x}{2 \sqrt{x}}+\frac{1}{\sqrt{x}}
\end{aligned}
$$

16) $f^{\prime}(x)=\ln x+\frac{x}{x}-1=\ln x+1-1=\ln x$.
17) $f^{\prime}(x)=\cos (\ln x) \cdot \frac{1}{x}=\frac{\cos (\ln x)}{x}$.
18) $f^{\prime}(x)=\frac{\left(\sin ^{2} x\right)^{\prime}}{\sin ^{2} x}=\frac{2 \sin x \cos x}{\sin ^{2} x}=\frac{2 \cos x}{\sin x}(=2 \cot x)$
19) $f^{\prime}(x)=\left(\ln x^{-1}\right)^{\prime}=-(\ln x)^{\prime}=-\frac{1}{x}$
20) 

$$
\begin{aligned}
y^{\prime}=\left[(\ln x)^{-1}\right]^{\prime} & =-(\ln x)^{-2} \cdot(\ln x)^{\prime} \\
& =-\frac{1}{(\ln x)^{2}} \cdot \frac{1}{x} \\
& =-\frac{1}{x(\ln x)^{2}}
\end{aligned}
$$

21) 

$$
\begin{aligned}
g^{\prime}(x) & =[\ln (a-x)-\ln (a+x)] \\
& =\frac{-1}{a-x}-\frac{1}{a+x}
\end{aligned}
$$

22) 

$$
\begin{aligned}
h^{\prime}(x) & =\frac{\left(x+\sqrt{x^{2}-1}\right)^{\prime}}{x+\sqrt{x^{2}-1}} \\
& =\frac{1+\frac{1}{2}\left(x^{2}-1\right)^{-\frac{1}{2}} \cdot 2 x}{x+\sqrt{x^{2}-1}}
\end{aligned}
$$

$$
=\frac{1+x\left(x^{2}-1\right)^{\frac{-1}{2}}}{x+\left(x^{2}-1\right)^{\frac{1}{2}}} .
$$

(You can multiply by $\frac{\left(x^{2}-1\right)^{\frac{1}{2}}}{\left(x^{2}-1\right)^{\frac{1}{2}}}$ to remove $(\cdots)^{-\frac{1}{2}}$. but this doesit simplify a lot.)
23)

$$
\begin{aligned}
G^{\prime}(y) & =\left[5 \ln (2 y+1)-\frac{1}{2} \ln \left(y^{2}+1\right)\right]^{\prime} \\
& =5 \cdot \frac{2}{2 y+1}-\frac{1}{2} \cdot \frac{2 y}{y^{2}+1} \\
& =\frac{10}{2 y+1}-\frac{y}{y^{2}+1} .
\end{aligned}
$$

24) 

$$
\begin{aligned}
f^{\prime}(u) & =\frac{(1+\ln u)-u\left(\frac{1}{u}\right)}{(1+\ln u)^{2}} \\
& =\frac{\ln u}{(1+\ln u)^{2}}
\end{aligned}
$$

26) 

$$
\begin{aligned}
H^{\prime}(z) & =\frac{1}{2}\left[\ln \left(a^{2}-z^{2}\right)-\ln \left(a^{2}+z^{2}\right)\right] \\
& =\frac{1}{2}\left[\frac{-2 z}{a^{2}-z^{2}}-\frac{2 z}{a^{2}+z^{2}}\right] \\
& =\frac{-z}{a^{2}-z^{2}}-\frac{z}{a^{2}+z^{2}} .
\end{aligned}
$$

27) 

$$
\begin{aligned}
f^{\prime}(u) & =\frac{(1+\ln (2 u)) \cdot \frac{1}{u}+\ln u \cdot\left(\frac{u}{2 u}\right)}{[1+\ln (2 u)]^{2}} \\
& =\frac{1}{u(1+\ln u)}+\frac{\ln u}{2(1+\ln (2 u))^{2}}
\end{aligned}
$$

28) $y^{\prime}=2 \ln (\tan x) \frac{\sec ^{2} x}{\tan x}$
$31)$

$$
\begin{aligned}
y^{\prime} & =\sec ^{2}(\ln (a x+b)) \cdot[\ln (a x+b)]^{\prime} \\
& =\sec ^{2}(\ln (a x+b)) \cdot \frac{a}{a x+b}
\end{aligned}
$$

32) 

$$
\begin{aligned}
y^{\prime} & =\frac{(\cos (\ln x))^{\prime}}{\cos (\ln x)} \\
& =\frac{-\sin (\ln x) \cdot \frac{1}{x}}{\cos (\ln x)} \\
& =-\frac{\tan (\ln x)}{x}
\end{aligned}
$$

33) 

$$
\begin{aligned}
y^{\prime} & =2 x \ln (2 x)+x^{2} \cdot \frac{2}{2 x}=2 x \ln (2 x)+x \\
y^{\prime \prime} & =2 \ln (2 x)+2 x \cdot \frac{2}{2 x}+1 \\
& =2 \ln (2 x)+3
\end{aligned}
$$

34) 

$$
\begin{aligned}
y^{\prime} & =\frac{\sec x \tan x+\sec ^{2} x}{\sec x+\tan x} \\
& =\sec x \\
\therefore y^{\prime \prime} & =\sec x \tan x
\end{aligned}
$$

38) 

$$
\begin{aligned}
& f^{\prime}(x)=\frac{1}{x^{2}}-\frac{\ln x}{x^{2}} \\
& f^{\prime \prime}(x)=-2 x^{-3}-\frac{1}{x^{3}}+\frac{2 \ln x}{x^{3}}=\frac{-3}{x^{3}}+\frac{2 \ln x}{x^{3}} \\
& \therefore f^{\prime \prime}(e)=-3 e^{-3}+2 e^{-3}=-e^{-3}
\end{aligned}
$$

42) 

$$
\begin{aligned}
& \ln (x y) \\
& \Rightarrow \quad y \sin x \\
& x y=y^{\prime} \sin x+y \cos x \\
& y+x y^{\prime}=x y \sin x \cdot y^{\prime}+x y^{2} \cos x \\
&(x-x y \sin x) y^{\prime}=x y^{2} \cos x-y \\
& \therefore y^{\prime}=\frac{x y^{2} \cos x-y}{x-x y \sin x}
\end{aligned}
$$

51) 

$$
\begin{aligned}
\ln y & =2 \ln \left(x^{2}+2\right)+4 \ln \left(x^{4}+4\right) \\
\therefore \frac{y^{\prime}}{y} & =2 \cdot \frac{2 x}{x^{2}+2}+4 \cdot \frac{4 x^{3}}{x^{4}+4} \\
& =\frac{4 x}{x^{2}+2}+\frac{16 x^{3}}{x^{4}+4} \\
y^{\prime} & =\left(x^{2}+2\right)^{2}\left(x^{4}+4\right)^{4}\left[\frac{4 x}{x^{2}+2}+\frac{16 x^{3}}{x^{4}+4}\right] \\
& =4 x\left(x^{2}+2\right)\left(x^{4}+4\right)^{4}+16 x^{3}\left(x^{2}+2\right)^{2}\left(x^{4}+4\right)^{3}
\end{aligned}
$$

52) omitted.
53) 

$$
\text { 54) } \begin{aligned}
\ln y & =4 \ln \left(x^{3}+1\right)+2 \ln \sin x-\frac{1}{3} \ln x \\
\therefore \frac{y^{\prime}}{y} & =4 \cdot \frac{3 x^{2}}{x^{3}+1}+2 \cdot \frac{\cos x}{\sin x}-\frac{1}{3 x} \\
& =\frac{12 x^{2}}{x^{3}+1}+\frac{2 \cos x}{\sin x}+\frac{1}{3 x} \\
\therefore y^{\prime}= & \frac{\left(x^{3}+1\right)^{4} \sin ^{2} x}{x^{\frac{1}{3}}}\left(\frac{12 x^{2}}{x^{3}+1}+\frac{2 \cos x}{\sin x}+\frac{1}{3 x}\right) \\
= & \frac{12\left(x^{3}+1\right)^{3} \sin ^{2} x}{x^{\frac{1}{3}}}+\frac{2\left(x^{3}+1\right)^{4} \sin x \cos x}{x^{\frac{1}{3}}}+\frac{\left(x^{3}+1\right)^{4} \sin ^{2} x}{3 x^{4 / 3}}
\end{aligned}
$$

55) Let $u=8-3 t \quad \therefore d u=-3 d t$

$$
\begin{aligned}
\int_{1}^{2} \frac{d t}{8-3 t} & =\frac{-1}{3} \int_{8-3 \cdot 1}^{8-3 \cdot 2} \frac{d u}{u} \\
& =\frac{-1}{3} \int_{5}^{2} \frac{d u}{u} \\
& =\frac{1}{3} \int_{2}^{5} \frac{d u}{u} \\
& =\frac{1}{3}[\ln |u|]_{2}^{5} \\
& =\frac{1}{3}(\ln 5-\ln 2)
\end{aligned}
$$

56) Omitted.
57) 

$$
\begin{aligned}
& \int_{4}^{9}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^{2} d x \\
= & \int_{4}^{9}\left(x+2+\frac{1}{x}\right) d x \\
= & {\left[\frac{x^{2}}{2}+2 x+\ln |x|\right]_{4}^{9} } \\
= & \frac{1}{2}\left(9^{2}-4^{2}\right)+2(9-4)+(\ln 9-\ln 4) \\
= & \frac{65}{2}+10+\ln \frac{9}{4} \\
= & \frac{85}{2}+\ln \frac{9}{4} .
\end{aligned}
$$

59) Let $u=\ln x \therefore d u=\frac{1}{x} d x$

$$
\int \frac{(\ln x)^{2}}{x} d x=\int u^{2} d u=\frac{u^{3}}{3}+c=\frac{(\ln x)^{3}}{3}+c .
$$

60) Let $u=\ln x \quad \therefore d u=\frac{1}{x} d x$

$$
\begin{aligned}
\int_{e}^{6} \frac{d x}{x \ln x}=\int_{\ln e}^{\ln 6} \frac{d u}{u} & =\int_{1}^{\ln 6} \frac{d u}{u} \\
& =[\ln |u|]_{1}^{\ln 6} \\
& =\ln (\ln 6) . \quad(\ln 6>0)
\end{aligned}
$$

Challenging
43)

$$
\begin{aligned}
& f^{\prime}(x)=\frac{1}{x-1}=(x-1)^{-1} \\
& f^{\prime \prime}(x)=-(x-1)^{-2} \\
& f^{(3)}(x)=2(x-1)^{-3}
\end{aligned}
$$

From this, it's not hard to see that

$$
f^{(n)}(x)=(-1)^{n-1}(n-1)!(x-1)^{-n}
$$

44) Let $f(x)=x^{8} \ln x$

$$
\begin{aligned}
\therefore f^{\prime}(x) & =8 x^{7} \ln x+x^{7} \\
f^{\prime \prime}(x) & =8 \cdot 7 x^{6} \ln x+8 x^{6}+7 x^{6}
\end{aligned}
$$

From this, it's easy to see that

$$
\begin{aligned}
f^{(8)}(x)= & 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \ln x \\
& +(\text { a polynomial of deg } 0, \text { ie. a constant) } \\
= & 8!\ln x+(\text { a constant }) \\
\therefore f^{(9)}(x)= & \frac{8!}{x} \quad(8!=8 \cdot 7 \cdots 2 \cdot 1)
\end{aligned}
$$

