

MTH 162 Homework 7

Do the first five problems. Due: Mar 5, 2014 (Wednesday). Hand in to me during the class.

Compulsory:

1–38 ■ Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$3. \lim_{x \rightarrow (\pi/2)^+} \frac{\cos x}{1 - \sin x}$$

$$4. \lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 5x}$$

$$5. \lim_{t \rightarrow 0} \frac{e^{2t} - 1}{\sin t}$$

$$6. \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$$

$$24. \lim_{x \rightarrow \infty} \sqrt{x} e^{-x/2}$$

Recommended: (These types of questions may also appear in the exams)

1–38 ■ Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

1. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - x}$

2. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

7. $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{1 + \cos 2\theta}$

8. $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{\csc \theta}$

9. $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$

10. $\lim_{x \rightarrow \infty} \frac{\ln \sqrt{x}}{x^2}$

11. $\lim_{t \rightarrow 1} \frac{t^8 - 1}{t^5 - 1}$

12. $\lim_{t \rightarrow 0} \frac{8^t - 5^t}{t}$

13. $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$

14. $\lim_{u \rightarrow \infty} \frac{e^{u/10}}{u^3}$

15. $\lim_{x \rightarrow 0} \frac{x3^x}{3^x - 1}$

16. $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$

17. $\lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$

18. $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(4x)}$

19. $\lim_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x - 1)^2}$

20. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

21. $\lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$

22. $\lim_{x \rightarrow a^+} \frac{\cos x \ln(x - a)}{\ln(e^x - e^a)}$

23. $\lim_{x \rightarrow 0} \cot 2x \sin 6x$

24. $\lim_{x \rightarrow \infty} \sqrt{x} e^{-x/2}$

25. $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$

26. $\lim_{x \rightarrow 0^+} \sin x \ln x$

27. $\lim_{x \rightarrow 1^+} \ln x \tan(\pi x/2)$

28. $\lim_{x \rightarrow \infty} x \tan(1/x)$

$$29. \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$$

$$30. \lim_{x \rightarrow 0} (\csc x - \cot x)$$

$$31. \lim_{x \rightarrow \infty} (x - \ln x)$$

$$32. \lim_{x \rightarrow 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)]$$

$$33. \lim_{x \rightarrow 0^+} x^{\sqrt{x}}$$

$$34. \lim_{x \rightarrow 0^+} (\tan 2x)^x$$

$$35. \lim_{x \rightarrow 0} (1 - 2x)^{1/x}$$

$$36. \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} \right)^{bx}$$

$$37. \lim_{x \rightarrow 1^+} x^{1/(1-x)}$$

$$38. \lim_{x \rightarrow \infty} (e^x + x)^{1/x}$$

Challenging (Attempt if you are interested)

41. Prove that

$$\lim_{x \rightarrow \infty} \frac{e^x}{x^n} = \infty$$

for any positive integer n . This shows that the exponential function approaches infinity faster than any power of x .

42. Prove that

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x .