MTH 162 Homework 3

Do the first four problems. Due: Feb 5, 2014 (Wednesday). Hand in to me during the class.

Compulsory:

Ex 5.3

23–36 ▪ Differentiate the function.

32. $y = \dfrac{e^u - e^{-u}}{e^u + e^{-u}}$

61–68 ▪ Evaluate the integral.

68. $\displaystyle\int_0^1 \dfrac{\sqrt{1 + e^{-x}}}{e^x}\, dx$

Ex 5.4

23–38 ▪ Differentiate the function.

32. $y = x^{\cos x}$

41–46 ▪ Evaluate the integral.

46. $\displaystyle\int \dfrac{2^x}{2^x + 1}\, dx$

Recommended: (These types of questions may also appear in the exams)

Ex 5.3

2–4 ▪ Simplify each expression.

2. (a) $e^{\ln 15}$

(b) $\ln(1/e)$

3. (a) $e^{-2 \ln 5}$

(b) $\ln\left(\ln e^{e^{10}}\right)$

4. (a) $\ln e^{\sin x}$

(b) $e^{x + \ln x}$

5–8 ▪ Solve each equation for x.

5. (a) $e^{7-4x} = 6$

(b) $\ln(3x - 10) = 2$

6. (a) $\ln(x^2 - 1) = 3$

(b) $e^{2x} - 3e^x + 2 = 0$

7. (a) $e^{3x+1} = k$

(b) $\ln x + \ln(x - 1) = 1$

8. (a) $\ln(\ln x) = 1$

(b) $e^{e^x} = 10$

23–36 ▪ Differentiate the function.

23. $f(x) = (x^3 + 2x)e^x$

24. $y = \dfrac{e^x}{1 - e^x}$

25. $y = e^{ax^3}$

26. $y = e^{-2t} \cos 4t$

27. $f(u) = e^{1/u}$

28. $y = x^2 e^{-1/x}$

29. $F(t) = e^{t \sin 2t}$

30. $y = e^{k \tan \sqrt{x}}$

33. $y = e^{e^x}$

34. $y = \sqrt{1 + xe^{-2x}}$

35. $y = \dfrac{ae^x + b}{ce^x + d}$

36. $f(t) = \sin^2(e^{\sin^2 t})$

Ex 5.3

61–68 ▪ Evaluate the integral.

61. $\displaystyle\int_0^2 \frac{dx}{e^{\pi x}}$

62. $\displaystyle\int x^2 e^{x^3}\,dx$

63. $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

64. $\displaystyle\int \frac{(1+e^x)^2}{e^x}\,dx$

65. $\displaystyle\int e^{\tan x}\sec^2 x\,dx$

66. $\displaystyle\int e^x\cos(e^x)\,dx$

67. $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

Ex 5.4

3–6 ▪ Write the expression as a power of e.

3. $4^{-\pi}$

4. $x^{\sqrt{5}}$

5. 10^{x^2}

6. $(\tan x)^{\sec x}$

23–38 ▪ Differentiate the function.

23. $f(x) = x^5 + 5^x$

24. $g(x) = x\sin(2^x)$

25. $f(t) = 10^{\sqrt{t}}$

26. $F(t) = 3^{\cos 2t}$

27. $L(v) = \tan(4^{v^2})$

28. $G(u) = (1 + 10^{\ln u})^6$

29. $y = 2x\log_{10}\sqrt{x}$

30. $y = \log_2(e^{-x}\cos \pi x)$

31. $y = x^x$

33. $y = x^{\sin x}$

34. $y = \sqrt{x}^{\,x}$

35. $y = (\cos x)^x$

36. $y = (\sin x)^{\ln x}$

37. $y = (\tan x)^{1/x}$

38. $y = (\ln x)^{\cos x}$

41–46 ▪ Evaluate the integral.

41. $\int_1^2 10^t \, dt$

42. $\int (x^5 + 5^x) \, dx$

43. $\int \frac{\log_{10} x}{x} \, dx$

44. $\int x 2^{x^2} \, dx$

45. $\int 3^{\sin\theta} \cos\theta \, d\theta$

Challenging: (Harder problems. Attempt if you are interested.)

Ex 5.3

72. Evaluate $\lim\limits_{x \to \pi} \dfrac{e^{\sin x} - 1}{x - \pi}$.

75. (a) Show that $e^x \geqslant 1 + x$ if $x \geqslant 0$.
     [*Hint:* Show that $f(x) = e^x - (1 + x)$ is increasing for $x > 0$.]
     (b) Deduce that $\frac{4}{3} \leqslant \int_0^1 e^{x^2} dx \leqslant e$.

76. (a) Use the inequality of Exercise 75(a) to show that, for $x \geqslant 0$,

$$e^x \geqslant 1 + x + \tfrac{1}{2}x^2$$

     (b) Use part (a) to improve the estimate of $\int_0^1 e^{x^2} dx$ given in Exercise 75(b).

(Hint: Consider integration e^t over the interval [1,x].)

Ex 5.4

49. Calculate $\lim_{x \to \infty} x^{-\ln x}$. (Hint: let $y = x^{-\ln x}$. What's the limit of y when x tends to infinity?)

54. Show that $\lim\limits_{n \to \infty} \left(1 + \dfrac{x}{n} \right)^n = e^x$ for any $x > 0$.

(Hint: Consider $(1 + \frac{x}{n})^n = \left((1 + \frac{x}{n})^{\frac{n}{x}} \right)^x$. For fixed x, what does this expression tend to when $n \to \infty$?)