MTH 162 Homework 11

Do the first four problems. Due: Apr 9, 2014 (Wednesday). Hand in to me during the class.

**Compulsory:**

**Ex 8.6**

**3–10** ▪ Find a power series representation for the function

**6.** $f(x) = \dfrac{1}{x + 10}$ (Hint: geometric series)

**Ex 8.7**

(In the following problems, write down the first four nonzero terms of the series if there is no obvious pattern. )

**5–10** ▪ Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. [Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$.](Maclaurin series=Taylor series centered at 0. )

**9.** $f(x) = \sinh x$

**11–18** ▪ Find the Taylor series for $f(x)$ centered at the given value of $a$. [Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$.]

**17.** $f(x) = \cos x, \quad a = \pi$

**43–46** ▪ Evaluate the indefinite integral as an infinite series.

**44.** $\displaystyle\int \frac{e^x - 1}{x}\,dx$

**Recommended:** (These types of questions may also appear in the exams)

**Ex 8.6**

**3–10** ▪ Find a power series representation for the function and determine the interval of convergence.

**3.** $f(x) = \dfrac{1}{1+x}$

**4.** $f(x) = \dfrac{5}{1-4x^2}$

**5.** $f(x) = \dfrac{2}{3-x}$

**6.** $f(x) = \dfrac{1}{x+10}$

**7.** $f(x) = \dfrac{x}{9+x^2}$

**8.** $f(x) = \dfrac{x}{2x^2+1}$

**9.** $f(x) = \dfrac{1+x}{1-x}$

**10.** $f(x) = \dfrac{x^2}{a^3-x^3}$

[In the exam, I won't ask you to find the radius of convergence.]

**15–20** ▪ Find a power series representation for the function and determine the radius of convergence.

**15.** $f(x) = \ln(5-x)$

**16.** $f(x) = x^2 \tan^{-1}(x^3)$

**17.** $f(x) = \dfrac{x}{(1+4x)^2}$

**18.** $f(x) = \left(\dfrac{x}{2-x}\right)^3$

**19.** $f(x) = \dfrac{1+x}{(1-x)^2}$

**20.** $f(x) = \dfrac{x^2+x}{(1-x)^3}$

**25–28** ▪ Evaluate the indefinite integral as a power series. What is the radius of convergence?

**25.** $\displaystyle\int \frac{t}{1-t^8}\,dt$

**26.** $\displaystyle\int \frac{t}{1+t^3}\,dt$

**27.** $\displaystyle\int x^2 \ln(1+x)\,dx$

**28.** $\displaystyle\int \frac{\tan^{-1}x}{x}\,dx$

## Ex 8.7

**5–10** ▪ Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. [Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$.] Also find the associated radius of convergence.

**5.** $f(x) = (1 - x)^{-2}$

**6.** $f(x) = e^{-2x}$

**7.** $f(x) = \sin \pi x$

**8.** $f(x) = x \cos x$

**9.** $f(x) = \sinh x$

**10.** $f(x) = \cosh x$

**11–18** ▪ Find the Taylor series for $f(x)$ centered at the given value of $a$. [Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$.]

**11.** $f(x) = x^4 - 3x^2 + 1, \quad a = 1$

**12.** $f(x) = x - x^3, \quad a = -2$

**13.** $f(x) = \ln x, \quad a = 2$

**14.** $f(x) = 1/x, \quad a = -3$

**15.** $f(x) = e^{2x}, \quad a = 3$

**16.** $f(x) = \sin x, \quad a = \pi/2$

**17.** $f(x) = \cos x, \quad a = \pi$

**18.** $f(x) = \sqrt{x}, \quad a = 16$

**27–36** ▪ Use a Maclaurin series in Table 1 to obtain the Maclaurin series for the given function.

**27.** $f(x) = \sin \pi x$

**28.** $f(x) = \cos(\pi x/2)$

**29.** $f(x) = e^x + e^{2x}$

**30.** $f(x) = e^x + 2e^{-x}$

**31.** $f(x) = x \cos(\frac{1}{2}x^2)$

**32.** $f(x) = x^2 \ln(1 + x^3)$

**33.** $f(x) = \dfrac{x}{\sqrt{4 + x^2}}$

**34.** $f(x) = \dfrac{x^2}{\sqrt{2 + x}}$

**35.** $f(x) = \sin^2 x$ [*Hint:* Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.]

**36.** $f(x) = \begin{cases} \dfrac{x - \sin x}{x^3} & \text{if } x \neq 0 \\ \frac{1}{6} & \text{if } x = 0 \end{cases}$

**43–46** ▪ Evaluate the indefinite integral as an infinite series.

**43.** $\int x \cos(x^3)\, dx$

**44.** $\int \frac{e^x - 1}{x}\, dx$

**45.** $\int \frac{\cos x - 1}{x}\, dx$

**46.** $\int \arctan(x^2)\, dx$

(For the following questions, the estimates of the error will not be tested in the exam. )

**47–50** ▪ Use series to approximate the definite integral to within the indicated accuracy.

**47.** $\int_0^1 x \cos(x^3)\, dx$ (three decimal places)

**48.** $\int_0^1 \sin(x^4)\, dx$ (four decimal places)

**49.** $\int_0^{0.1} \frac{dx}{\sqrt{1 + x^3}}$ $(|\text{error}| < 10^{-8})$

**50.** $\int_0^{0.5} x^2 e^{-x^2}\, dx$ $(|\text{error}| < 0.001)$

---

**51–53** ▪ Use series to evaluate the limit.

**51.** $\lim_{x \to 0} \frac{x - \ln(1 + x)}{x^2}$

**52.** $\lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x}$

**53.** $\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$