MTH 162 Homework 1

Do the first four problems. Due: Jan 22, 2014 (Wednesday). Hand in to me during the class.

Compulsory:

Ex 5.1

21–26 ▪ Find a formula for the inverse of the function.

23. $f(x) = 1 + \sqrt{2 + 3x}$

37–40 ▪ Find $(f^{-1})'(a)$.

38. $f(x) = x^3 + 3 \sin x + 2 \cos x, \quad a = 2$

Ex 5.2

1–4 ▪ Use the Laws of Logarithms to expand the quantity.

2. $\ln \sqrt[3]{\dfrac{x-1}{x+1}}$

5–8 ▪ Express the quantity as a single logarithm.

6. $\ln 3 + \frac{1}{3} \ln 8$ (Please fully simplify your answer)

Recommended: (These types of questions may also appear in the exams)

Ex 5.1

17. If $h(x) = x + \sqrt{x}$, find $h^{-1}(6)$.

21–26 ▪ Find a formula for the inverse of the function.

22. $f(x) = \dfrac{4x - 1}{2x + 3}$

24. $y = 2x^3 + 3$

25. $y = \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$

26. $f(x) = 2x^2 - 8x, \quad x \geqslant 2$

37–40 ▪ Find $(f^{-1})'(a)$.

37. $f(x) = 2x^3 + 3x^2 + 7x + 4, \quad a = 4$

39. $f(x) = 3 + x^2 + \tan(\pi x/2), \quad -1 < x < 1, \quad a = 3$

40. $f(x) = \sqrt{x^3 + x^2 + x + 1}, \quad a = 2$

Ex. 5.2

1–4 ▪ Use the Laws of Logarithms to expand the quantity.

1. $\ln \sqrt{ab}$

4. $\ln s^4 \sqrt{t\sqrt{u}}$

5–8 ▪ Express the quantity as a single logarithm.

5. $\ln 5 + 5 \ln 3$

7. $\frac{1}{3} \ln(x + 2)^3 + \frac{1}{2} [\ln x - \ln(x^2 + 3x + 2)^2]$

8. $\ln(a + b) + \ln(a - b) - 2 \ln c$

Challenging: (Harder problems. Attempt if you are interested.)

Ex. 5.1

31. Let $f(x) = \sqrt{1 - x^2}$, $0 \leqslant x \leqslant 1$.
(a) Find f^{-1}. How is it related to f?
(b) Identify the graph of f and explain your answer to part (a).

43. If $f(x) = \int_3^x \sqrt{1 + t^3}\, dt$, find $(f^{-1})'(0)$. (Hint: what's $a=f^{-1}(0)$? Use the 2nd fundamental theorem of calculus.)

48. (a) If f is a one-to-one, twice differentiable function with inverse function g, show that

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}$$

(b) Deduce that if f is increasing and concave upward, then its inverse function is concave downward.

Ex 5.2

69. By comparing areas, show that

$$\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < \ln n < 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}$$

73. Use the definition of derivative to prove that

$$\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$$

(Hint: ln 1=0, how's $\lim_{x \to 0} \frac{\ln(1 + x)}{x}$ related to the derivative of ln?)