Math 311 — Practice Test 3

(1) (a) Give the differential equation describing the motion of an object of mass $m = 2$ attached to a spring (with spring constant $k = 8$) that is not damped, and solve the equation.
   (b) Now connect the mass to a dashpot (with dampening constant b) to dampen the system. What dampening constant $b = b_{crit}$ causes the system to be critically damped? Give the general solution to this differential equation.
   (c) Then solve the equation for each situation $b = 2b_{crit}$ and $b = \frac{1}{2}b_{crit}$. Which one is overdamped and which is underdamped?

(2) The motion of a damped mass-spring system experiencing an external periodic force is given by $x'' + 2x' + 26x = 600\cos 10t$.
   (a) Given the solution $x(t)$ satisfying the initial conditions $x(0) = 10$, $x'(0) = 0$.
   (b) Identify both the transient and steady periodic components of the solution. That is, write $x(t) = x_{tr}(t) + x_{sp}(t)$ where $\lim_{t\to\infty} x_{tr}(t) = 0$ and $x_{sp}(t)$ is periodic.
   (c) Write $x_{sp}(t)$ is amplitude-phase form.

(3) For each of the following systems of linear differential equations
   (a) Give the general solution.
   (b) Sketch the phase portraits.
   (c) Give the solution satisfying $x(0) = 0, y(0) = 1$ and include/identify the corresponding curve in the phase portrait.

$$\begin{cases} x' &= 3x + y \\ y' &= 5x - y \end{cases} \qquad \begin{cases} x' &= 5x + 6y \\ y' &= -2x - 2y \end{cases} \qquad \begin{cases} x' &= -2y \\ y' &= 5x - 2y \end{cases}$$