Math 311 — Practice Test 3

(1) For each of the following systems of linear differential equations
  (a) Give the general solution.
  (b) Sketch the phase portraits.
  (c) Give the solution satisfying $x(0) = 0, y(0) = 1$ and include/identify the corresponding curve in the phase portrait.

(2) Solve the each of the systems of linear differential equations and sketch their phase portraits:

- $\begin{cases} x' &= 5x + 6y \\ y' &= -2x - 2y \end{cases}$
- $\begin{cases} x' &= 3x + y \\ y' &= 3y \end{cases}$
- $\begin{cases} x' &= 3x + y \\ y' &= 5x - y \end{cases}$
- $\begin{cases} x' &= -2y \\ y' &= 5x - 2y \end{cases}$

(3) For the non-linear system $\begin{cases} x' &= -x + xy \\ y' &= 3y - y^2 - xy \end{cases}$
  (a) Find the equilibrium points.
  (b) Sketch the phase portrait of the linearization of the system at each equilibrium point.
  (c) Sketch the phase portrait of the non-linear system. (Use the previous info and nullclines.)
  (d) Based on your sketch of the phase portrait: If $(x(t), y(t))$ is a solution curve for the non-linear system with initial conditions $(x(0), y(0)) = (3, 5)$, what is $\lim_{t\to\infty} (x(t), y(t))$?

(4) The non-linear system $\begin{cases} x' &= x^2 - 2x - xy \\ y' &= y^2 - 4y + xy \end{cases}$ models the population of a predator-prey relationship.
  (a) Which variable describes the predator population? the prey population?
  (b) Find the equilibrium points.
  (c) Sketch the phase portrait of the linearization of the system at each equilibrium point.
  (d) Sketch the phase portrait of the non-linear system. Use the previous info and nullclines.
  (e) Discuss the possible the long term behaviors of these populations.

(5) Solve the following initial value problems.
  (a) $x'' + 9x = 0; \quad x(0) = 3, x'(0) = 4$
  (b) $x'' - 4x = 3t; \quad x(0) = x'(0) = 0$
  (c) $x'' + 4x' + 5x = 5e^{-2t}; \quad x(0) = x'(0) = 0$
  (d) $x^{(4)} + x = 0; \quad x(0) = x'(0) = x''(0) = 0, x^{(3)}(0) = 1$
  (e) $x'' + 4x = E(t); \quad x(0) = 10, x'(0) = 0,$ and $E(t) = \begin{cases} 1 \text{ if } t \leq 1 \\ 0 \text{ if } t > 1 \end{cases}$
  (f) $x'' + 9x = \delta_4(t); \quad x(0) = x'(0) = 1$
  (g) $\begin{cases} 0 &= x' + 2y' + x \\ 0 &= x' - y' + y \end{cases}; \quad x(0) = 0, y(0) = 1$

(6) Let $f(t) = t - 2$ when $2 \leq t \leq 3$ and let $f(t) = 0$ otherwise.
  (a) Compute the Laplace transform of $f(t)$ directly from the definition of the Laplace transform.
  (b) Compute the Laplace transform by first expressing $f(t)$ in terms of fundamental functions, step functions, and translations and then using the properties of the Laplace transform.