Math 311 — Practice Test 2

CHAPTER 2

(1) For the autonomous differential equation $\frac{dx}{dt} = x^2 - 4x + 3$:
  (a) Find all critical point and draw a phase diagram. For each critical point, determine if it is stable, unstable, or semi-stable.
  (b) If $x(t)$ is a solution to the IVP $x_0 = x(0)$, determine $\lim_{t\to\infty} x(t)$ in terms of x_0.
  (c) Sketch several solution curves on an appropriate domain.
  (d) Give a general solution to the differential equation. (Partial fraction decompositions help here, but that won't be on this exam.)

(2) For the autonomous differential equation $\frac{dx}{dt} = x^2(e^{2x-3} - 1)$:
  (a) Find all critical point and draw a phase diagram. For each critical point, determine if it is stable, unstable, or semi-stable.
  (b) If $x(t)$ is a solution to the IVP $x_0 = x(0)$, determine $\lim_{t\to\infty} x(t)$ in terms of x_0.
  (c) Sketch several solution curves on an appropriate domain.

(3) Make a differential equation that mathematically models the spread of a rumor in the situation described below. Determine the relevant domains for your variables. Qualitatively describe how the rumor may spread depending on initial conditions.

> In a large university with a fixed population of people, the rate of change of the number of those people who have heard a certain rumor is proportional to the number that have not yet heard the rumor.

(4) An object moving horizontally experiences resistance due to friction that is:
  (a) proportional to the square root of its speed (absolute value of velocity) and
  (b) in the direction opposite its motion.

  If there are no other forces contributing to its horizontal motion, obtain an equation for its velocity $v(t)$ at time t with initial velocity $v(0) = v_0 > 0$.
  Also obtain an equation for its position $x(t)$ with initial position $x(0) = x_0$.

CHAPTER 3

(5) How many solutions are there to the IVP $y'' + \cos(x)y' + \frac{1}{1+x^2}y = 0$ where $y(0) = 2$ and $y'(0) = -1$? What is the domain of each solution?

(6) Show that $y_1 = x$ and $y_2 = x \ln x$ are linearly independent solutions to the differential equation $x^2y'' - xy' + y = 0$. Give a general solution. Then find the solution that satisfies the initial conditions $y(1) = 7$ and $y'(1) = 2$.

(7) Give a general solution to $2y''' + 3y'' + 2y' = 0$.

(8) Give general solutions to the following differential equations:
  - $y'' - 4y' + 4y = \sin(2x)$
  - $y^{(4)} - 4y'' + 4y = 6e^{2t}$

(9) Give a differential equation that has $y = 3xe^{-x} + 2x^2 \cos(x/3)$ as a solution.

(10) Solve the IVP $x'' + 2x' - 8x = -3te^{2t}, x(0) = 1, x'(0) = 0$.

(11) Solve the differential equation $Ly = 7e^{-3t}$
  where L is the linear differential operator $L = (D^2 - 9)D$.