Math 210 — Practice Test 3

(1) (a) Find the determinant of the matrix $A = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 3 & 1 & 0 & -1 \end{bmatrix}$.

(b) Is A invertible? If so, what is $\det(A^{-1})$? If not, why not?

(2) Quickly, what is the determinant of $A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -\frac{7}{8} & 6 \\ 3 & 1 & 0 & 42 & \frac{238}{19938} & \pi & 0 \\ 0 & -1 & 28 & 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & 1 & -2 & 2 & 9 \\ 0 & 0 & 0 & 1 & 10 & -7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 93 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$?

(3) What numbers arise as determinants of orthogonal matrices? Justify your answer.

(4) (a) How can you easily conclude that the matrix $B = \begin{bmatrix} 3 & 1 & 33 & 8 \\ 9 & -2 & 99 & 1 \\ -7 & 5 & -77 & 11 \\ 2 & 1 & 22 & -8 \end{bmatrix}$ is singular?

(b) Why does this tell you that 4 is an eigenvalue of $A = \begin{bmatrix} 7 & 1 & 33 & 8 \\ 9 & 2 & 99 & 1 \\ -7 & 5 & -73 & 11 \\ 2 & 1 & 22 & -4 \end{bmatrix}$?

(c) Use this to find an eigenvector of A with eigenvalue 4. Then find a unit eigenvector of A with eigenvalue 4.

(5) (a) What are the eigenvalues of $A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$?

(b) Find an eigenvector for each eigenvalue; describe the eigenspace for each eigenvalue.

(c) Use this information to diagonalize A.

(d) Repeat this for the matrix $M = \begin{bmatrix} 3 & 2 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

(6) The matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ is symmetric. Find an *orthonormal basis* for $\mathbb{R}^3$ consisting of eigenvectors of A. Use this to construct a diagonalization $A = Q\Lambda Q^T$.

(7) The Singular Value Decomposition of $A = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 2 & -2 & 0 & 2 \end{bmatrix}$ is:

$$\underset{A}{\begin{bmatrix} 1 & 0 & -1 & -1 \\ 2 & -2 & 0 & 2 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \underset{\Sigma}{\begin{bmatrix} 2\sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{3} & 0 & 0 \end{bmatrix}} \underset{V^T}{\begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{3} & 0 & 1/\sqrt{3} \\ 1/\sqrt{3} & 0 & -1/\sqrt{3} & -1/\sqrt{3} \\ 1/\sqrt{6} & 2/\sqrt{6} & 0 & 1/\sqrt{6} \\ 1/\sqrt{6} & 0 & 2/\sqrt{6} & -1/\sqrt{6} \end{bmatrix}}$$

(a) What are the eigenvalues of A^TA? What are the eigenvalues of AA^T?

(b) What is the closest rank 1 approximation to A?

(c) Using this SVD, give an orthonormal basis for the null space of A.

(d) What is the pseudoinverse A^+ of A?

(8) Let P_2 be the vector space of polynomials of degree at most 2. That is, $P_2 = \{ax^2+bx+c | a, b, c \in \mathbb{R}\}$.

(a) Show that the transformation $T\colon P_2 \to P_2$ that takes a polynomial $p(x)$ to the polynomial $p(x-1)$ is a linear transformation.

(b) Choose a basis for P_2 and find the associated matrix of this linear transformation.